Date: 12/10/2021

Q1. Find the distance between the points (2, – 3) and (–6, 3).

Q2. Find the distance between the points (0, 0) and (3, 4).

Q3. Find the distance between the points (2, 3) and (1, – 4).

Q4. Find the distance between the points $(a - b, a + b)$ and $(a + b, a - b)$.

Q5. Find the distance between the points $(\cos\alpha, -\sin\alpha)$ and $(-\cos\alpha, \sin\alpha)$

Q6. Find the distance between the points $(a\cos\alpha, a\sin\alpha)$ and $(a\cos\beta, a\sin\beta)$.

Q7. Show that the following points are the vertices of an isosceles triangle $O(0, 0)$, $A(8, 2)$ and $B(5, -3)$.

Q8. The vertices of a triangle ABC are $A(1, 1)$, $B(2, 3)$ and $C(5, 7)$ find $\cos A$.

Q9. Show that the following points are the vertices of a right-angled isosceles triangle $A(7, 9)$, $B(3, -7)$ and $C(-3, 3)$.

Q10. Show that the following points represent a rectangle $O(0, 0)$, $A(0, 5)$, $B(6, 5)$ and $C(6, 0)$.

Q11. Show that the following points represent a square $A(3, 2)$, $B(0, 5)$, $C(-3, 2)$ and $D(0, -1)$.

Q12. The opposite angular points of a square are (2, 0) and (5, 1). Find the remaining points.

Q13. Draw a quadrilateral in the cartesian plane, whose vertices are (– 4, 5) (0, 7), (5, – 5) and (– 4, – 2). Also, find its area.

Q14. Find a point on the x-axis, which is equidistant from the points (7, 6) and (3, 4).

Q15. The base of an equilateral triangle with side $2a$ lies along the y-axis such that the mid-point of the base is at the origin. Find vertices of the triangle.

Q16. Find the distance between $P(x_1, y_1)$ and $Q(x_2, y_2)$ when : (i) PQ is parallel to the y-axis, (ii) PQ is parallel to the x-axis.

Q17. Two vertices of an equilateral triangle are (0, 0) and $(3, \sqrt{3})$. Find the third vertex.

Date: 12/10/2021

S1. Let A (2, – 3) and B (– 6, 3) be the points then

$$AB = \sqrt{(-6-2)^2 + \{3-(-3)\}^2}$$

$$= \sqrt{(-8)^2 + (3+3)^2} = \sqrt{(-8)^2 + 6^2}$$

$$= \sqrt{64+36} = \sqrt{100} = 10 \text{ units.}$$

S2. Let A (0, 0) and B (3, 4) be points then

$$AB = \sqrt{(0-3)^2 + (0-4)^2}$$

$$= \sqrt{9+16} = \sqrt{25} = 5 \text{ units}$$

S3. Let A (2, 3) and B (1, – 4)

$$AB = \sqrt{(1-2)^2 + (-4-3)^2}$$

$$= \sqrt{1+49} = 5\sqrt{2}$$

S4. Let A ($a – b$, $a + b$) and B ($a + b$, $a – b$)

$$AB = \sqrt{[(a+b)-(a-b)]^2 + [(a-b)-(a+b)]^2}$$

$$= \sqrt{(2b)^2 + (-2b)^2}$$

$$= \sqrt{8b^2} = 2\sqrt{2}\,b$$

S5. Let A ($\cos\alpha$, $-\sin\alpha$) and B ($-\cos\alpha$, $\sin\alpha$)

$$AB = \sqrt{(-\cos\alpha - \cos\alpha)^2 + (\sin\alpha + \sin\alpha)^2}$$

$$= \sqrt{4\cos^2\alpha + 4\sin^2\alpha}$$

$$= \sqrt{4(\cos^2\alpha + \sin^2\alpha)}$$

$$= \sqrt{4(1)} = 2$$

S6. Let A ($a\cos\alpha$, $a\sin\alpha$) and ($a\cos\beta$, $a\sin\beta$)

$$AB = \sqrt{a^2(\cos\beta - \cos\alpha)^2 + a^2(\sin\beta - \sin\alpha)^2}$$

$$= a\sqrt{\cos^2\beta + \cos^2\alpha - 2\cos\alpha\cos\beta + \sin^2\beta + \sin^2\alpha - 2\sin\alpha\sin\beta}$$

$$= a\sqrt{2 - 2(\sin\alpha\sin\beta + \cos\alpha\cos\beta)}$$

$$= a\sqrt{2 - 2\cos(\alpha - \beta)} = a\sqrt{2[1 - \cos(\alpha - \beta)]}$$

$$= a\sqrt{2 \cdot 2\sin^2\frac{\alpha - \beta}{2}} = 2a\sin\frac{\alpha - \beta}{2}.$$

S7. Let $O(0, 0)$, $A(8, 2)$, $B(5, -3)$

$$OA = \sqrt{(8-0)^2 + (2-0)^2} = \sqrt{68}$$

$$OB = \sqrt{(5-0)^2 + (-3-0)^2} = \sqrt{34}$$

$$AB = \sqrt{(5-8)^2 + (-3-2)^2} = \sqrt{34}$$

As, $OB = AB$, the triangle is isosceles.

S8. Let, $a = BC = \sqrt{(5-2)^2 + (7-3)^2} = \sqrt{25} = 5$

$$b = AC = \sqrt{(5-1)^2 + (7-1)^2}$$

$$= \sqrt{52} = 2\sqrt{13}$$

and $c = AB = \sqrt{(2-1)^2 + (3-1)^2} = \sqrt{5}$

Applying cosine rule,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{52 + 5 - 25}{2(2\sqrt{13})(\sqrt{5})} = \frac{8}{\sqrt{65}}.$$

S9. Let $A(7, 9)$, $B(3, -7)$ and $C(-3, 3)$

$$AB^2 = (3-7)^2 + (-7-9)^2 = 16 + 256 = 272$$

$$BC^2 = (-3-3)^2 + (3+7)^2 = 36 + 100 = 136$$

$$AC^2 = (-3-7)^2 + (3-9)^2 = 100 + 36 = 136$$

Now, $BC^2 + AC^2 = 136 + 136 = 272 = AB^2$

By Pythagoras theorem triangle is right angled and $AC = BC \Rightarrow$ triangle is isosceles.

S10. Let $O(0, 0)$, $A(0, 5)$, $B(6, 5)$ and $C(6, 0)$

$$OA = \sqrt{(0-0)^2 + (5-0)^2} = 5,$$

$$BC = \sqrt{(6-6)^2 + (0-5)^2} = 5$$

$$AB = \sqrt{(6-0)^2 + (5-5)^2} = 6$$

$$OC = \sqrt{(6-0)^2 + (0-0)^2} = 6$$

$$OB = \sqrt{(6-0)^2+(5-0)^2} = \sqrt{51}$$

Now, $OA^2 + AB^2 = (5)^2 + (6)^2 = 51 = OB^2$

As, opposite sides are equal and one angle is 90°, it is a rectangle.

S11. Let $A\,(3, 2)$, $B\,(0, 5)$, $C\,(-3, 2)$ and $D\,(0, -1)$.

$$AB = \sqrt{(0-3)^2+(5-2)^2} = \sqrt{18}$$

$$BC = \sqrt{(-3-0)^2+(2-5)^2} = \sqrt{18}$$

$$CD = \sqrt{(0+3)^2+(-1-2)^2} = \sqrt{18}$$

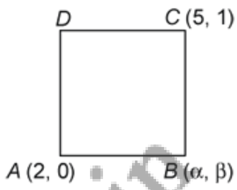
$$DA = \sqrt{(3-0)^2+(2+1)^2} = \sqrt{18}$$

Now, $AC^2 = \sqrt{(-3-3)^2+(2-2)^2} = 36$

$$AB^2 + BC^2 = 18 + 18 = 36 = AC^2$$

Since, all the sides are equal and one angle is 90°, hence it is square.

S12. Let the unknown vertex be $B\,(\alpha, \beta)$.



We know that

$$AB^2 = BC^2$$

$\Rightarrow$ $(\alpha-2)^2 + (\beta-0)^2 = (\alpha-5)^2 + (\beta-1)^2$

$\Rightarrow$ $\alpha^2 - 4\alpha + 4 + \beta^2 = \alpha^2 - 10\alpha + 25 + \beta^2 - 2\beta + 1$

$\Rightarrow$ $6\alpha + 2\beta = 22$

$\Rightarrow$ $\beta = 11 - 3\alpha$ ... (i)

As, $B = 90°$, we get

$$AB^2 + BC^2 = AC^2$$

$\Rightarrow$ $(\alpha-2)^2 + (\beta-0)^2 + (\alpha-5)^2 + (\beta-1)^2 = (5-2)^2 + (1-0)^2$

$\Rightarrow$ $\alpha^2 - 4\alpha + 4 + \beta^2 + \alpha^2 - 10\alpha + 25 + \beta^2 - 2\beta + 1 = 10$

$\Rightarrow$ $2\alpha^2 + 2\beta^2 - 14\alpha - 2\beta + 20 = 0$

$\Rightarrow$ $\alpha^2 + \beta^2 - 7\alpha - \beta + 10 = 0$

$\Rightarrow$ $\alpha^2 + (11-3\alpha)^2 - 7\alpha - (11-3\alpha) + 10 = 0$ [From (i)]

$\Rightarrow$ $\alpha^2 + 9\alpha^2 + 121 - 66\alpha - 7\alpha - 11 + 3\alpha + 10 = 0$

$\Rightarrow$ $10\alpha^2 - 70\alpha + 120 = 0$

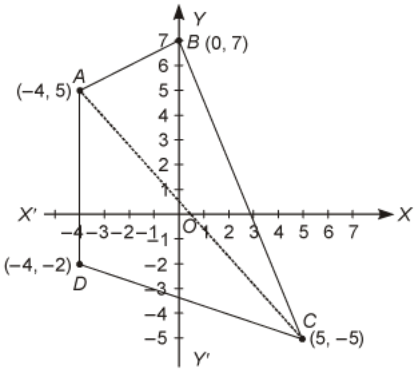
$\Rightarrow$ $\alpha^2 - 7\alpha + 12 = 0 \Rightarrow \alpha = 4, 3$

From (i)

when $\alpha = 4$, $\beta = -1$, when $\alpha = 3$, $\beta = 2$

Therefore, the required vertices are $(4, -1)$ and $(3, 2)$.

S13. Join AC. Now we have two triangles, ΔABC and ΔACD.



$$\text{Area } (\Delta ABC) = \frac{1}{2}|-4(7+5)+0(-5-5)+5(5-7)|$$

$$= \frac{1}{2}|-48+0-10| = \frac{1}{2}\times 58 = 29.$$

$$\text{Area } (\Delta ACD) = \frac{1}{2}|-4(-5+2)+5(-2-5)-4(5+5)|$$

$$= \frac{1}{2}|12-35-40| = \frac{63}{2} = 31.5.$$

Now, Area of quad. $ABCD$ = 29 + 31.5 = 60.5 sq. units.

S14. Let the point on x-axis be $P(x, 0)$

Let A be (7, 6) and B be (3, 4)

Now, $$PA = \sqrt{(x-7)^2+(0-6)^2}$$

$$PA^2 = (x-7)^2 + 36$$

and $$PB = \sqrt{(x-3)^2+(0-4)^2}$$

$$PB^2 = (x-3)^2 + 16$$

Given $$PA = PB$$

or $$PA^2 = PB^2$$

or $$(x-7)^2 + 36 = (x-3)^2 + 16$$

or $$x^2 - 14x + 49 + 36 = x^2 + 9 - 6x + 16$$

or $$-14x + 6x = 25 - 85$$

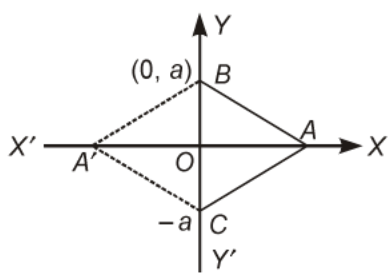
or $$-8x = -60$$

or $$8x = 60$$

or $$2x = 15 \quad \Rightarrow \quad x = \frac{15}{2}$$

Hence, the required point is $\left(\frac{15}{2}, 0\right)$.

S15. A.T.Q., two triangles ΔABC and $\Delta A'BC$ are possible.



Given $BC = 2a$ and mid-point of BC is at (0, 0).

$\Rightarrow$ $$OB = OC = a$$

i.e., coordinates of B and C are $(0, a)$ and $(0, -a)$, respectively.

As triangles are equilateral, we have in ΔABC

$$AB = BC = CA = 2a$$

Applying Pythagoras theorem

$$OA = \sqrt{AB^2 - OB^2}$$
$$= \sqrt{(2a)^2 - a^2}$$
$$= \sqrt{4a^2 - a^2} = \sqrt{3a^2} = \sqrt{3}a$$

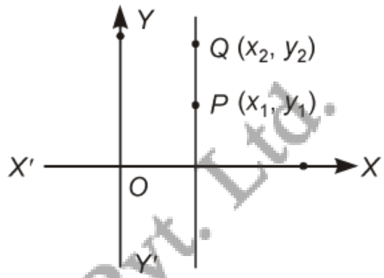
Similarly, $OA' = \sqrt{3}a$

As A and A' lie on X-axis, coordinates of A and A' are $(\sqrt{3}a, 0)$ and $(-\sqrt{3}a, 0)$, respectively.

Vertices of $\Delta ABC = (0, a), (0, -a), (\sqrt{3}a, 0)$

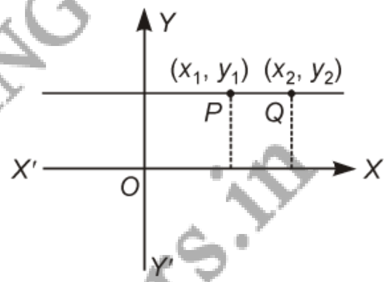
Vertices of $\Delta A'BC = (0, a), (0, -a), (-\sqrt{3}a, 0)$.

S16. (i) In this case, $x_1 = x_2$



Now,
$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{0 + (y_2 - y_1)^2}$$
$$= |y_2 - y_1|.$$

(ii) In this case, $y_1 = y_2$



Now,
$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(x_2 - x_1)^2 + 0}$$
$$= |x_2 - x_1|.$$

S17. Let the third vertex be $P(x, y)$ and let $O(0, 0)$ and $A(3, \sqrt{3})$.

Now, $OP = OA = AP$

$$OP^2 = OA^2 = AP^2,$$

$$OP^2 = OA^2$$

$\Rightarrow$ $x^2 + y^2 = (0 - 3)^2 + (0 - \sqrt{3})^2$

$\Rightarrow$ $x^2 + y^2 = 12$ ... (i)

$$OP^2 = AP^2$$

$\Rightarrow$ $x^2 + y^2 = (3 - x)^2 + (\sqrt{3} - y)^2$

$$= 9 + x^2 - 6x + 3 + y^2 - 2\sqrt{3}y$$

$\Rightarrow$ $6x + 2\sqrt{3}y = 12$ ... (ii)

From Eq. (i) and (ii), we get

$$x^2 + \left[\frac{6 - 3x}{\sqrt{3}}\right]^2 = 12$$

$\Rightarrow \quad 4x^2 - 12x = 0$

$\Rightarrow \quad 4x(x - 3) = 0$

$\Rightarrow \quad x = 0, 3$

From (ii), $y = \dfrac{6 - 3x}{\sqrt{3}}$ for $x = 0$, $y = 2\sqrt{3}$

for $x = 3$, $y = -\sqrt{3}$

Hence, the point is $(0, 2\sqrt{3})$ or $(3, -\sqrt{3})$

Date: 12/10/2021

Q1. A line passes through (x_1, y_1) and (h, k). If slope of the line is m, show that $k - y_1 = m(h - x_1)$.

Q2. Find the value of x for which the points $(x, -1)$, $(2, 1)$ and $(4, 5)$ are collinear.

Q3. Show that the following points are collinear (1, 3), (2, 5) and (4, 9).

Q4. For what value of k are the three lines $2x - 5y + 3 = 0$, $5x - 9y + k = 0$ and $x - 2y + 1 = 0$ conccurent?

Q5. Find the area of a triangle whose vertices are (2, 3), (4, – 1) and (7, 0).

Q6. Prove that the following points are collinear. Also, find the equation of the line.

(1, 3), (3, 5) and (5, 7)

Q7. If $a \neq b \neq c$, prove that the points (a, a^2), (b, b^2) and (c, c^2) can never be collinear.

Q8. Show that the lines $x - y = 6$, $4x - 3y = 20$ and $6x + 5y + 8 = 0$ are concurrent. Also find their point of intersection.

Q9. Find the area of the triangle formed by the lines $y - x = 0$, $x + y = 0$ and $x - k = 0$.

Q10. If the points (a, b), (a_1, b_1) and $(a - a_1, b - b_1)$ are collinear show that $ab_1 = a_1b$. Also show that the line joining the given points passes through the origin.

Q11. If the point $(a, 0)$, $(0, b)$ and $(1, 1)$ are collinear, show that $\frac{1}{a} + \frac{1}{b} = 1$.

Q12. Show that the following sets of these lines are concurrent $\frac{x}{a} + \frac{y}{b} = 1$, $\frac{x}{b} + \frac{y}{a} = 1$ and $x = y$.

Q13. By using the concept of equation of a line, prove that the three points (3, 0), (– 2, – 2) and (8, 2) are collinear.

Q14. If the lines $2x + y - 3 = 0$, $5x + ky - 3 = 0$ and $3x - y - 2 = 0$ are concurrent, find the value of k.

Q15. If lines whose equations are $y = m_1x + c_1$, $y = m_2x + c_2$ and $y = m_3x + c_3$ meet in a point then prove that $m_1(c_2 - c_3) + m_2(c_2 - c_1) + m_3(c_1 - c_2) = 0$

Q16. For what values of k are the three lines $4x + 7y - 9 = 0$, $5x + ky + 15 = 0$ and $9x - y + 6 = 0$ are concurrent.

Q17. The coordinates of points A, B and C are (1, 2), (– 2, 1) and (0, 6). Verify that the medians of the triangle ABC are concurrent. Also, find the co-ordinates of the point of concurrency (centroid).

Q18. Prove that the following lines are concurrent:

$5x - 3y = 1, \; 2x + 3y = 23, \; 42x + 21y = 257$

Date: 12/10/2021

S1. Let the points be $A(x_1, y_1)$ and $B(h, k)$.

Now, $$m_{AB} = \frac{k - y_1}{h - x_1}$$

and $$m_{AB} = m$$

Hence, $$\frac{k - y_1}{h - x_1} = m$$

or $$k - y_1 = m(h - x_1)$$

which is the required result.

S2. Let the points be $A(x, -1)$, $B(2, 1)$ and $C(4, 5)$.

Now as the points are collinear, therefore

$$m_{AB} = m_{BC}$$

Now, $$m_{AB} = \frac{1+1}{2-x} = \frac{2}{2-x}$$

and $$m_{BC} = \frac{5-1}{4-2} = \frac{4}{2} = 2$$

As $$m_{AB} = m_{BC}$$

We have $$\frac{2}{2-x} = 2$$

$$2 = 2(2 - x) = 4 - 2x$$

or $$2x = 4 - 2$$

$\Rightarrow$ $$2x = 2 \Rightarrow x = 1.$$

S3. If the points are collinear, the area of the triangle formed by them will be zero.

Area of a triangle is given by $$A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 2 & 5 & 1 \\ 4 & 9 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[1(5-9) - 3(2-4) + 1(18-20)]$$

$$= \frac{1}{2}[-4+6-2] = 0$$

Hence the points are collinear.

S4.

$$2x - 5y + 3 = 0 \quad \ldots \text{(i)}$$

$$5x - 9y + k = 0 \quad \ldots \text{(ii)}$$

$$x - 2y + 1 = 0 \quad \ldots \text{(iii)}$$

Solving Eqs. (i) and (iii) simultaneously, we get

$$\frac{x}{-5+6} = \frac{y}{3-2} = \frac{1}{-4+5}, \quad \frac{x}{1} = \frac{y}{1} = \frac{1}{1} \quad \Rightarrow \quad \boxed{x=1} \text{ and } \boxed{y=1}$$

Since the lines are concurrent, these values of x and y will satisfy Eq. (ii)

$$5 \times 1 - 9 \times 1 + k = 0, \quad 5 - 9 + k = 0 \quad \Rightarrow \quad \boxed{k=4}.$$

S5.

Area of a triangle is given by $A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$$A = \frac{1}{2}\begin{vmatrix} 2 & 3 & 1 \\ 4 & -1 & 1 \\ 7 & 0 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[2(-1-0) - 3(4-7) + 1(0+7)]$$

$$= \frac{1}{2}[-2+9+7] = 7 \text{ square units.}$$

S6. A (1, 3), B (3, 5) and C (5, 7)

$$\text{Slope of } AB = m_1 = \frac{5-3}{3-1} = 1$$

$$\text{Slope of } AC = m_2 = \frac{7-3}{5-1} = 1$$

As $m_1 = m_2$, $AB \parallel AC \Rightarrow A, B, C$ are collinear.

Applying point slope form, we get

$$y - x - 3 = -1 \quad \Rightarrow \quad y = x + 2.$$

which is the required equation.

S7.

Area of a triangle is given by $A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

We find area of the triangle formed by the given points

$$\text{Area} = \frac{1}{2}\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[a(b^2 - c^2) - a^2(b - c) + 1(bc^2 - b^2c)]$$

$$= \frac{1}{2}[a(b - c)(b + c) - a^2(b - c) - bc(b - c)]$$

$$= \frac{1}{2}(b - c)[a(b + c) - a^2 - bc]$$

$$= \frac{1}{2}(b - c)(a - b)(c - a)$$

And this will never be zero because $a \neq b, \quad b \neq c, \quad c \neq a$

Hence the points are never collinear.

S8. The given equations are:

$$x - y - 6 = 0 \qquad \dots \text{(i)}$$

$$4x - 3y - 20 = 0 \qquad \dots \text{(ii)}$$

$$6x + 5y + 8 = 0 \qquad \dots \text{(iii)}$$

On solving (i) and (ii) by cross multiplication, we get

$$\frac{x}{(20 - 18)} = \frac{y}{(-24 + 20)} = \frac{1}{(-3 + 4)}$$

$$\Rightarrow \qquad \frac{x}{2} = \frac{y}{-4} = \frac{1}{1} \quad \Rightarrow \quad x = 2, \; y = -4.$$

Thus, the lines (i) and (ii) intersect at the point $P(2, -4)$.

Putting $x = 2$ and $y = -4$ in (iii), we get

$$\text{L.H.S.} = 6 \times 2 + 5 \times (-4) + 8 = 0 = \text{R.H.S.}$$

This shows that the point $P(2, -4)$ also lies on (iii).

Thus, all the given three lines intersect at the same point.

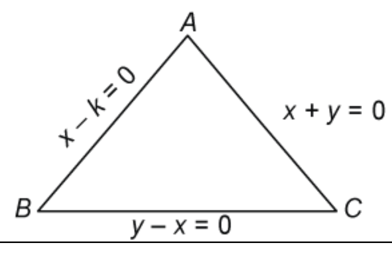
Hence, the given lines are concurrent and their point of intersection is $P(2, -4)$.

S9. Let ABC be the triangle whose sides are

$$AB : x - k = 0 \qquad \dots \text{(i)}$$

$$BC : y - x = 0 \qquad \dots \text{(ii)}$$

$$AC : x + y = 0 \qquad \dots \text{(iii)}$$



Solving (i) and (iii), we get the coordinates of $A : (k, -k)$

Solving (i) and (ii), we get the coordinates of B : (k, k)

Solving (ii) and (iii), we get the coordinates of C : (0, 0).

$\therefore$ Area of triangle $ABC = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

$$= \frac{1}{2}\{k(k-0) + k(0+k) + 0(-k-k)\}$$

$$= \frac{1}{2}\{k^2 + k^2\} = \frac{1}{2} \times 2k^2 = k^2$$

S10. If the points are collinear, the area of the triangle formed by them must be zero.

$$\frac{1}{2}\begin{vmatrix} a & b & 1 \\ a_1 & b_1 & 1 \\ a-a_1 & b-b_1 & 1 \end{vmatrix} = 0$$

$\Rightarrow \quad a[b_1 - (b - b_1)] - b[a_1 - (a - a_1)] + [a_1(b - b_1) - b_1(a - a_1)] = 0$

$\Rightarrow \quad a[2b_1 - b] - b[2a_1 - a] + [a_1b - b_1a] = 0$

$\Rightarrow \quad ab_1 - a_1b = 0$ ... (i)

Further, if the origin (0, 0) lies on the line joining the points (a, b) and (a_1, b_1), we must get

$$\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ a & b & 1 \\ a_1 & b_1 & 1 \end{vmatrix} = 0$$

$\Rightarrow \quad [0 + 0 + 1\,(ab_1 - a_1b)] = 0$

$\Rightarrow \quad ab_1 = a_1b$

which is true from (i).

S11. If the points are collinear, the area of the triangle formed by them is zero.

Area of a triangle is given by $A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

i.e., $\frac{1}{2}\begin{vmatrix} a & 0 & 1 \\ 0 & b & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$

$\Rightarrow \quad a[b-1] + 0 + 1[0-b] = 0$

$\Rightarrow \quad ab - a - b = 0$

$\Rightarrow \quad a + b = ab \quad \Rightarrow \quad \frac{1}{a} + \frac{1}{b} = 1.$

S12.

$$\frac{x}{a}+\frac{y}{b} = 1 \quad \Rightarrow \quad bx + ay - ab = 0 \qquad \text{... (i)}$$

$$\frac{x}{b}+\frac{y}{a} = 1 \quad \Rightarrow \quad ax + by - ab = 0 \qquad \text{... (ii)}$$

$$x = y \quad \Rightarrow \quad x - y = 0 \qquad \text{... (iii)}$$

$$\Rightarrow \qquad \frac{x}{-ab\,(b-a)} = \frac{y}{-ab\,(b-a)} = \frac{1}{(b-a)(b+a)}$$

$$\Rightarrow \qquad x = \frac{-ab}{a+b} \text{ and } y = \frac{-ab}{a+b}$$

Put these values in Eq. (iii), we get L.H.S. = R.H.S.

Hence the given lines are concurrent.

S13. The given points are: $A\,(3, 0)$, $B\,(-2, -2)$ and $C\,(8, 2)$.

First we find equation of AB which is

$$y - y_1 = \frac{y_1 - y_2}{x_1 - x_2}(x - x_1)$$

or $$y - 0 = \frac{0+2}{3+2}(x-3)$$

or $$5y = 2x - 6$$

Now, let $C\,(8, 2)$ lies on it.

Therefore, $$5 \times 2 = 2 \times 8 - 6$$

$$10 = 10 \text{ which is true.}$$

Thus, C lies on the equation of AB.

Hence, A, B and C are colliear.

S14. Three lines are said to be concurrent, if they pass through a common point i.e., point of intersection of any two lies lies on the third line. Here given lines are

$$2x + y - 3 = 0 \qquad \text{... (i)}$$

$$5x + ky - 3 = 0 \qquad \text{... (ii)}$$

$$3x - y - 2 = 0 \qquad \text{... (iii)}$$

Soving Eq. (i) and (iii) by cross-multiplication method, we get

$$\frac{x}{-2-3} = \frac{y}{-9+4} = \frac{1}{-2-3} \quad \text{or} \quad x = 1, \; y = 1$$

Therefore, the point of intersection of two lines is (1, 1). Since above three lines are concurrent, the point (1, 1) will satisfy Eq. (ii) so that

$$5 \cdot 1 + k \cdot 1 - 3 = 0 \quad \text{or} \quad k = -2.$$

S15. The equations of the given lines are

$$m_1x - y + c_1 = 0 \quad \text{... (i)}$$

$$m_2x - y + c_2 = 0 \quad \text{... (ii)}$$

$$m_3x - y + c_3 = 0 \quad \text{... (iii)}$$

Solving (i) and (ii), we get

$$\frac{x}{-c_2 + c_1} = \frac{y}{m_2c_1 - m_1c_2} = \frac{1}{-m_1 + m_2}$$

Thus, the point of intersection of (i) and (ii) is

$$\left(\frac{c_1 - c_2}{m_2 - m_1}, \frac{m_2c_1 - m_1c_2}{m_2 - m_1}\right)$$

The three lines will be concurrent if the point of intersection of (i) and (ii) lies on (iii).

$$\Rightarrow \quad m_3\left(\frac{c_1 - c_2}{m_2 - m_1}\right) - \left(\frac{m_2c_1 - m_1c_2}{m_2 - m_1}\right) + c_3 = 0$$

$$m_3(c_1 - c_2) - (m_2c_1 - m_1c_2) + c_3(m_2 - m_1) = 0$$

$$\Rightarrow \quad m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0.$$

S16. The given lines are

$$4x + 7y - 9 = 0 \quad \text{... (i)}$$

$$5x + ky + 15 = 0 \quad \text{... (ii)}$$

$$9x - y + 6 = 0 \quad \text{... (iii)}$$

Solving (i) and (iii) simultaneously, we get

$$\frac{x}{42 - 9} = \frac{y}{-81 - 24} = \frac{1}{-4 - 63}$$

$$x = -\frac{33}{67}, \qquad y = \frac{105}{67}$$

Thus, the point of intersection of (i) and (iii) is $\left(-\frac{33}{67}, \frac{105}{67}\right)$.

Since, the three lines are concurrent, this point of intersection lies on (ii), *i.e.*, $5x + ky + 15 = 0$.

$$\Rightarrow \quad 5\left(-\frac{33}{67}\right) + k\left(\frac{105}{67}\right) + 15 = 0$$

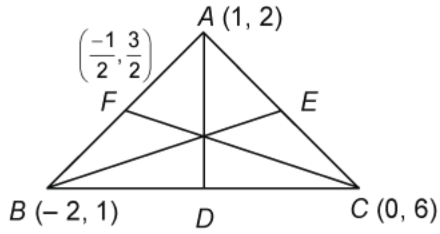
$$\Rightarrow \quad -165 + 105k + 1005 = 0$$

$$\Rightarrow \quad 105k = -840 \quad \Rightarrow \quad k = \frac{-840}{105}$$

Hence, for $k = -8$, the given lines are concurrent.

S17. We have the co-ordinates of A, B and C are (1, 2), (– 2, 1) and (0, 6) respectively. Let D and E be the mid-points of BC and AC respectively.

$\therefore \qquad D = \left(\frac{-2+0}{2}, \frac{1+6}{2}\right)$ and $E = \left(\frac{1+0}{2}, \frac{2+6}{2}\right)$



$$D = \left(-1, \frac{7}{2}\right), \quad E = \left(\frac{1}{2}, 4\right)$$

Equation of median AD passing through $A(1, 2)$ and $D\left(-1, \frac{7}{2}\right)$ is

$$y - 2 = \frac{\frac{7}{2} - 2}{-1-1}(x - 1) \quad \Rightarrow \quad y - 2 = -\frac{3}{4}(x - 1)$$

$$\Rightarrow \qquad 3x + 4y - 11 = 0 \qquad \text{... (i)}$$

Equation of median BE passing through $B(-2, 1)$ and $E\left(\frac{1}{2}, 4\right)$ is

$$y - 1 = \frac{4-1}{\frac{1}{2}+2}(x + 2) \quad \Rightarrow \quad y - 1 = \frac{6}{5}(x+2)$$

$$\Rightarrow \qquad 6x - 5y + 17 = 0 \qquad \text{... (ii)}$$

On solving (i) and (ii), we get the co-ordinates of the point of intersection $\left(-\frac{1}{3}, 3\right)$.

Similarly, Eq. of C.F. $(y - 0) = \dfrac{0+\frac{1}{2}}{6-\frac{3}{2}}\left(x + \frac{1}{2}\right)$

$$\Rightarrow \qquad y = \frac{1}{9}\left(x + \frac{1}{2}\right) \quad \Rightarrow \quad 2x - 9y + 1 = 0 \qquad \text{... (iii)}$$

Since, $\left(-\frac{1}{3}, 3\right)$ lies on Eq. (iii), hence medians of the triangle ABC are concurrent.

S18. Given lines are:

$$5x - 3y - 1 = 0 \qquad \text{... (i)}$$

$$2x + 3y - 23 = 0 \qquad \text{... (ii)}$$

$$42x + 21y - 257 = 0 \qquad \text{... (iii)}$$

Solving (i) and (ii), we get

$$\frac{x}{69+3} = \frac{y}{-2+115} = \frac{1}{15+6}$$

$$\Rightarrow \qquad \frac{x}{72} = \frac{y}{113} = \frac{1}{21} \quad \Rightarrow \quad x = \frac{72}{21} \text{ and } y = \frac{113}{21}$$

Thus, the lines (i) and (ii) intersect at the point $\left(\frac{72}{21}, \frac{113}{21}\right)$.

$\left(\frac{72}{21}, \frac{113}{21}\right)$ lies on (iii) if $42\left(\frac{72}{21}\right) + 21\left(\frac{113}{21}\right) - 257 = 0$

$\Rightarrow$ if $2 \times 72 + 113 - 257 = 0$ $\Rightarrow$ if $144 + 113 - 257 = 0$

$\Rightarrow$ $257 - 257 = 0$, which is true.

Thus, $\left(\frac{72}{21}, \frac{113}{21}\right)$ is on all the lines. Hence, given lines are concurrent.

Date: 12/10/2021

Q1. Determine whether the lines joining the given points are parallel, perpendicular or neither

A (1, 4), B (3, 6), C (– 1, 5) and D (2, 8).

Q2. Determine whether the lines joining A (3, 2), B (4, 5), C (2, 3) and D (5, 2) points are parallel, perpendicular or neither.

Q3. Prove that the given lines are parallel: $x + 3y + 4 = 0$ and $2x + 6y - 7 = 0$.

Q4. Prove that the given lines are perpendicular: $2x + 3y + 3 = 0$ and $3x - 2y + 5 = 0$.

Q5. Find the slope of a line, which passes through the origin, and the mid-point of the line segment joining the points $P(0, -4)$ and $B(8, 0)$.

Q6. Find the value of A for which the following lines are parallel.

$2x + 3y = 7$ and $Ax + 6x = 8$

Q7. Without using the Pythagoras theorem, show that the points (4, 4), (3, 5) and (– 1, – 1) are the vertices of a right angled triangle.

Q8. Find the equation of the line which satisfy the given conditions: Passing through (0, 0) with slope m.

Q9. Find the equation of the line which satisfy the given conditions: Intersecting the x-axis at a distance of 3 units to the left of origin with slope – 2.

Q10. Find the equation of the line which satisfy the given conditions: Intersecting the y-axis at a distance of 2 units above the origin and making an angle of 30° with positive direction of the x-axis.

Q11. Find the equation of the line passing through (– 3, 5) and perpendicular to the line passing through the points (2, 5) and (– 3, 6).

Q12. Find the equation of the line (i) passing through (– 2, 3) with slope – 4, (ii) which makes intercepts – 3 and 2 on the x- and y-axes respectively.

Q13. Find the distance of the line $4x - y = 0$ from the point P (4, 1) measured along the line making an angle of 135 with the positive x-axis.

Q14. Without using Pythagoras theorem, show that the following points represent the vertices of right triangle

A (1, 3), B (3, 5) and C (– 3, 7).

Q15. Three points A (x_1, y_1), B (x_2, y_2) and C (x, y) are collinear. Prove that

$$(x - x_1)(y_2 - y_1) = (x_2 - x_1)(y - y_1).$$

Q16. Determine whether the lines joining the given below points are parallel, perpendicular or neither

A (3, 7), B (1, 4), C (– 2, 4) and D (0, 3).

Q17. Find the slope of the line which makes an angle 30° with the positive direction of y-axis measured anticlockwise.

Q18. Find the slope of a line through the points:

(a) (1, 2) and (4, 2) (b) (0, – 4) and (– 6, 2)

Q19. Show that the line joining the points (2, – 3) and (– 5, 1) is parallel to the line joining the points (7, – 1) and (0, 3) and perpendicular to the line joining (4, 5) and (0, – 2).

Q20. Find the equation of the line which satisfy the given conditions: Passing through $(2, 2\sqrt{3})$ and inclined with the x-axis at an angle of 75°.

Q21. Consider the following population and year graph (see figure), find the slope of the line AB and using it, find what will be the population in the year 2010.

Q22. Find the angle between the x-axis and the line joining the points (3, – 1) and (4, – 2).

Q23. Without using distance formula, show that points (– 2, – 1), (4, 0), (3, 3) and (– 3, 2) are the vertices of a parallelogram.

Q24. The slope of a line is double of the slope of another line. If tangent of the angle between them is $\frac{1}{3}$, find the slope of the lines.

Q25. Find the values of k for which the line $(k - 3)x - (4 - k^2)y + (k^2 - 7k + 6) = 0$ is:

(i) Parallel to the x-axis (ii) Parallel to the y-axis (iii) Passing through the origin.

Q26. If angle between the lines is $\frac{\pi}{4}$ and slope of one of the lines is $\frac{1}{2}$, find the slope of the other line.

Q27. Find the slope of the lines:

(i) passing through the points (3, – 2) and (– 1, 4).

(ii) passing through the points (3, – 2) and (7, – 2).

(iii) passing through the points (3, – 2) and (3, 4).

(iv) Making inclination of 60° with positive direction of x-axis.

Q28. A line through the points (– 2, 6) and (4, 8) is perpendicular to the line through the points (8, 12) and (x, 24). Find the value of x.

Q29. Write the equation of the lines for which $\tan\theta = \frac{1}{2}$, where θ is the inclination of the line and (i) y-intercept is $-\frac{3}{2}$ (ii) x-intercept is 4.

Q30. Find the equation of a line perpendicular to the line $x - 2y + 3 = 0$ and passing through the point (1, – 2).

Q31. Show that two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where $b_1, b_2 \neq 0$ are: (i) Parallel if $\frac{a_1}{b_1} = \frac{a_2}{b_2}$, and (ii) Perpendicular if $a_1a_2 + b_1b_2 = 0$.

Q32. A quadrilateral has the vertices at the points (– 4, 2), (2, 6), (8, 5) and (9, – 7). Show that the mid points of the sides of this quadrilateral are the vertices of a parallelogram.

Q33. Find the coordinates of the orthocentre of the triangle with vertices

A (10, 4), B (– 2, – 1) and C (– 4, 9)

Date: 12/10/2021

S1. A (1, 4), B (3, 6), C (– 1, 5), D (2, 8)

$$m_1 = \text{slope of } AB = \frac{6-4}{3-1} = 1,$$

$$m_2 = \text{slope of } CD = \frac{8-5}{2+1} = 1$$

As $m_1 = m_2$, the lines are parallel.

S2. A (3, 2), B (4, 5), C (2, 3), D (5, 2)

$$m_1 = \text{slope of } AB = \frac{5-2}{4-3} = 3$$

$$m_2 = \text{slope of } CD = \frac{2-3}{5-2} = -\frac{1}{3}$$

As $m_1 m_2 = -1$, the lines are perpendicular.

S3. The slope of the line $x + 3y = -4$ is $m_1 = -\frac{1}{3}$

The slope of the line $2x + 6y = 7$ is $m_2 = -\frac{1}{3}$

As $m_1 = m_2$, the lines are parallel.

S4. The slope of the line $2x + 3y + 3 = 0$ is $m_1 = -\frac{2}{3}$

The slope of the line $3x - 2y + 5 = 0$ is $m_2 = \frac{3}{2}$

$$m_1 m_2 = -\frac{2}{3} \cdot \frac{3}{2} = -1$$

Therefore, the lines are perpendicular.

S5. Co-ordinates of mid-point of the line segment joining the points P(0, – 4) and B (8, 0) are $\left(\frac{8+0}{2}, \frac{0-4}{2}\right)$ *i.e.*, (4, – 2).

Now, slope of the line passing through origin (0, 0) and (4, – 2).

$$= \frac{-2-0}{4-0} = \frac{-2}{4} = -\frac{1}{2}$$

S6. If the lines are parallel, we must get $\frac{A}{2} = \frac{6}{3} \Rightarrow A = 4$.

S7. Let the vertices be $A(4, 4)$, $B(3, 5)$ and $C(-1, -1)$.

Slope of AB, $m_1 = \frac{5-4}{3-4} = \frac{1}{-1} = -1$

Slope of BC, $m_2 = \frac{-1-5}{-1-3} = \frac{-6}{-4} = \frac{3}{2}$

Slope of AC, $m_3 = \frac{-1-4}{-1-4} = 1$

Now, $m_1 m_3 = (-1) \times (1) = -1$

Thus, $AB \perp AC$ or $\angle A = 90°$

Hence, ΔABC is a right angled triangle.

S8. The line passes through (0, 0) and its slope = m.

Equation of the line is $y - y_1 = m(x - x_1)$

$\Rightarrow$ $y - 0 = m(x - 0)$

$\Rightarrow$ $y = mx$.

S9. The line passes through $(-3, 0)$ and has slope -2.

Now, equation of the line in point-slope form is

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -2(x + 3)$$

$$y = -2x - 6$$

$$2x + y + 6 = 0$$

which is the required equation.

S10. The required line passes through (0, 2) and makes an angle of 30° with positive direction of x-axis.

Hence, $m = \tan 30° = \frac{1}{\sqrt{3}}$

Now, equation of the line in point-slope form is

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{1}{\sqrt{3}}(x - 0)$$

$$\sqrt{3}y - 2\sqrt{3} = x$$

$$x - \sqrt{3}y + 2\sqrt{3} = 0.$$

S11. The required line passes through $A(-3, 5)$ and is perpendicular to the line passing through the points $B(2, 5)$ and $C(-3, 6)$.

$$m_{BC} = \frac{6-5}{-3-2} = -\frac{1}{5}$$

Slope of required line = 5.

$$y - y_1 = m(x - x_1)$$

$$y - 5 = 5(x + 3)$$

$$y - 5 = 5x + 15$$

$$5x - y + 20 = 0.$$

S12. (i) Here, $m = -4$ and given point (x_0, y_0) is $(-2, 3)$.

Equation of the given line is

$$y - 3 = -4(x + 2)$$

or $$4x + y + 5 = 0$$

which is the required equation.

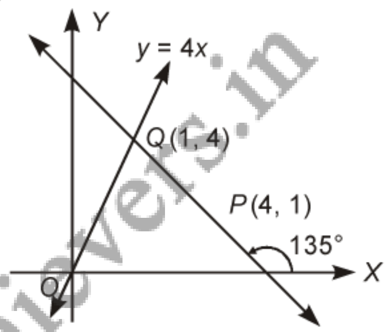
(ii) Here $a = -3$ and $b = 2$. Equation of the line is

$$\frac{x}{-3} + \frac{y}{2} = 1 \quad \text{or} \quad 2x - 3y + 6 = 0.$$

S13. Given line is $$4x - y = 0 \qquad \text{... (i)}$$

In order to find the distance of the line in Eq. (i) from the point $P(4, 1)$ along another line, we have to find the point of intersection of both the lines. For this purpose, we will first find the equation of the second line (see figure). Slope of second line is $\tan 135° = -1$. Equation of the line with slope -1 through the point $P(4, 1)$ is

$$y - 1 = -1(x - 4) \quad \text{or} \quad x + y - 5 = 0 \qquad \text{... (ii)}$$



Sloving Eq. (i) and (ii), we get $x = 1$ and $y = 4$, so that point of intersection of the two lines is $Q(1, 4)$. Now, distance of line in Eq. (i) from the point $P(4, 1)$ along the line in Eq. (ii)

= The distance between the points $P(4, 1)$ and is $Q(1, 4)$

$$= \sqrt{(1-4)^2 + (4-1)^2} = 3\sqrt{2} \text{ units.}$$

S14. Let, $A(1, 3)$, $B(3, 5)$ and $C(-3, 7)$

$$\text{Slope of } AB = m_1 = \frac{5-3}{3-1} = 1.$$

$$\text{Slope of } BC = m_2 = \frac{7-5}{-3-3} = -\frac{1}{3}$$

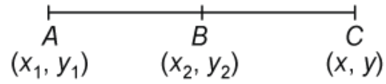
$$\text{Slope of } AC = m_3 = \frac{7-3}{-3-1} = -1$$

Now, $m_1 m_3 = -1 \quad \Rightarrow \quad AB \perp AC$

$\Rightarrow$ Δ is right angled

S15. Since, Three points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x, y)$ are collinear. Therefore,

Slope of AB = Slope of AC



$$\Rightarrow \quad \frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1} \quad \Rightarrow \quad (x - x_1)(y_2 - y_1) = (x_2 - x_1)(y - y_1).$$

S16. $A(3, 7)$, $B(1, 4)$, $C(-2, 4)$ and $D(0, 3)$

$$m_1 = \text{slope of } AB = \frac{4-7}{1-3} = \frac{3}{2}$$

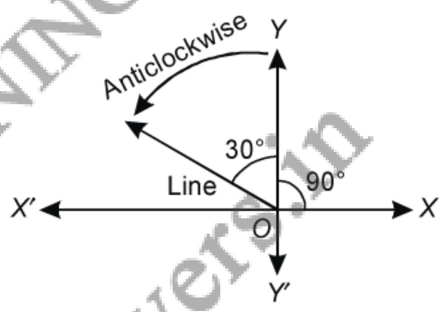
$$m_2 = \text{slope of } CD = \frac{3-4}{0+2} = -\frac{1}{2}$$

As $m_1 \neq m_2$, and $m_1 m_2 \neq -1$

the lines are intersecting (neither parallel nor perpendicular).

S17. The line makes 30° with y-axis when measured anticlockwise.

$\Rightarrow$ The line makes (90° + 30°) *i.e.*, 120° with the x-axis as shown in the figure.



$$\text{Slope of the line} = \tan 120°$$

$$= -\sqrt{3}$$

S18. (a) Here, $x_1 = 1$, $y_1 = 2$, $x_2 = 4$, $y_2 = 2$

We know that,

$$\text{Slope of line joining two points} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\therefore \quad \text{Required slope} = \frac{2-2}{4-1} = \frac{0}{3} = 0$$

(b) Here, $x_1 = 0$, $y_1 = -4$, $x_2 = -6$ and $y_2 = 2$

$$\text{Slope of the line joining } (0, -4) \text{ and } (-6, 2) = \frac{2+4}{-6-0} = \frac{6}{-6} = -1.$$

S19. Let m_1 be the pole of line l_1 joining points $(2, -3)$ and $(-5, 1)$.

$$\therefore \quad m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1+3}{-5-2} = -\frac{4}{7} \qquad \ldots \text{(i)}$$

Let m_2 be the pole of line l_2 joining points (7, – 1) and (0, 3).

$$\therefore \quad m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1+3}{0-7} = -\frac{4}{7} \quad \text{... (ii)}$$

Let m_3 be the pole of line l_3 joining points (4, 5) and (0, – 2).

$$\therefore \quad m_3 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-5}{0-4} = \frac{4}{7} \quad \text{... (iii)}$$

From Eqs. (i) and (ii), we observe that

$$m_1 = -\frac{4}{7} = m_2$$

Thus, line l_1 and l_2 are parallel.

From Eqs. (i) and (iii), we observe that

$$m_1 \cdot m_3 = \left(-\frac{4}{7}\right)\left(\frac{7}{4}\right) = -1$$

Thus, line l_1 and l_3 are perpendicular.

S20. The line passes through $(2, 2\sqrt{3})$

$$\theta = 75°$$

$$m = \tan 75° = \tan (45° + 30°)$$

$$= \frac{1+\tan 30°}{1-\tan 30°}$$

$$= \frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$y - y_1 = m(x - x_1)$$

$$y - 2\sqrt{3} = \frac{\sqrt{3}+1}{\sqrt{3}-1}(x-2)$$

or $\quad \sqrt{3}x - 2\sqrt{3} + x - 2 = \sqrt{3}y - 6 - y + 2\sqrt{3}$

$$\sqrt{3}x + x - \sqrt{3}y + y - 2\sqrt{3} - 2\sqrt{3} - 2 + 6 = 0$$

$$(\sqrt{3} + 1)x - (\sqrt{3} - 1)y = 4\sqrt{3} - 4$$

or $\quad (\sqrt{3} + 1)x - (\sqrt{3} - 1)y = 4(\sqrt{3} - 1)$

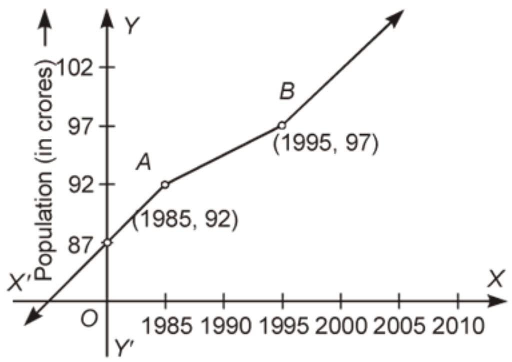
S21. A(1985, 92), B(1995, 97).

$$\text{Slope of } AB = m = \frac{97-92}{1995-1985} = \frac{5}{10} = \frac{1}{2}$$

Let population for 2010 be y crores.

Thus, C is the point having coordinates (2010, y).

Now, slope of BC



$$\Rightarrow \quad \frac{y-97}{2010-1995} = \frac{1}{2}$$

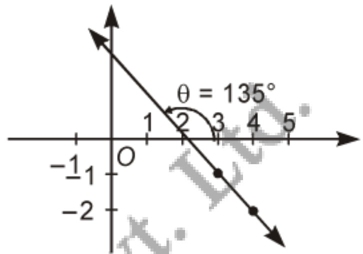
$$\Rightarrow \quad 2(y-97) = 15$$

$$\Rightarrow \quad y - 97 = 7.5$$

$$y = 104.5$$

Hence, population in 2010 will be 104.5 crores.

S22. Let the given points be $A(3, -1)$ and $B(4, -2)$.



Here, $$m_{AB} = \frac{-2+1}{4-3} = \frac{-1}{1} = -1$$

Also, slope of x-axis, $m = 0$.

Let θ be the angle between these two lines.

Then, $$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

or $$\tan\theta = \frac{0+1}{1+0} = 1$$

or $$\theta = 45°$$

Now, obtuse angle = 180° – 45° = 135°.

Hence, the angle between x-axis and the given line is 135°.

S23. Let the vertices be $A(-2, -1)$, $B(4, 0)$, $C(3, 3)$ and $D(-3, 2)$.

Now, $$m_{AB} = \frac{0+1}{4+2} = \frac{1}{6}$$

$$m_{BC} = \frac{3-0}{3-4} = -3$$

$$m_{CD} = \frac{2-3}{-3-3} = \frac{-1}{-6} = \frac{1}{6}$$

$$m_{AD} = \frac{2+1}{-3+2} = -3$$

Now, $m_{AB} = m_{CD}$ = $AB \parallel CD$

and $m_{BC} = m_{AD}$ = $BC \parallel AD$

Hence, $ABCD$ is a parallelogram.

S24. If slope of one line is m. Then the slope of the other line is $2m$.

Let angle between these two lines be θ. Then, $\tan\theta = \frac{1}{3}$

But, $\tan\theta = \frac{\tan\alpha_2 - \tan\alpha_1}{1+\tan\alpha_1 \tan\alpha_2}$ $\Rightarrow$ $\frac{1}{3} = \frac{2m-m}{1+m(2m)}$

$\Rightarrow$ $\frac{1}{3} = \frac{m}{1+2m^2}$ $\Rightarrow$ $1 + 2m^2 = 3m$

$\Rightarrow$ $2m^2 - 3m + 1 = 0$ $\Rightarrow$ $(2m-1)(m-1) = 0$

$\Rightarrow$ $2m - 1 = 0$ or $m - 1 = 0$

$\Rightarrow$ $m = \frac{1}{2}$ or $m = 1$

Thus the slope of these lines are $\frac{1}{2}$ and 1.

S25. Given equation is

$$(k-3)x - (4-k^2)y + (k^2 - 7k + 6) = 0$$

Slope of the given equation is

$$m_1 = \frac{(k-3)}{(4-k^2)} = \frac{k-3}{4-k^2}$$

(i) Also slope of x-axis, $m_2 = 0$

As they are parallel,

$$\frac{k-3}{4-k^2} = 0 \Rightarrow k = 3$$

(ii) Now, slope of y-axis, $m = \frac{1}{0}$

Hence, $\frac{k-3}{4-k^2} = \frac{1}{0}$

$\Rightarrow$ $4 - k^2 = 0 \Rightarrow k = \pm 2$

(iii) Origin (0, 0) lies on the line,

$$(k-3)x - (4-k^2)y + (k^2 - 7k + 6) = 0$$

or $(k-3)\times 0 - (4-k^2)\times 0 + k^2 - 7k + 6 = 0$

$\Rightarrow$ $k^2 - 7k + 6 = 0$

$\Rightarrow$ $(k-6)(k-1) = 0$

$\Rightarrow$ $k = 1, 6.$

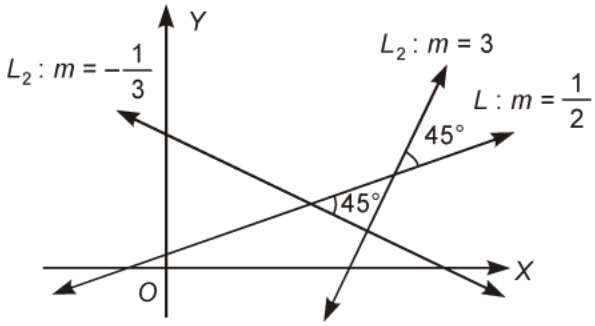
S26. We know that the acute angle θ between two lines with slopes m_1 and m_2 is gven by

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| \quad \text{... (i)}$$

Let $m_1 = \frac{1}{2}$, $m_2 = m$ and $\theta = \frac{\pi}{4}$

Now, putting these values in Eq. (i), we get

$$\tan\frac{\pi}{4} = \left|\frac{m - \frac{1}{2}}{1 + \frac{1}{2}m}\right| \quad \text{or} \quad 1 = \left|\frac{m - \frac{1}{2}}{1 + \frac{1}{2}m}\right|$$



which gives $\frac{m - \frac{1}{2}}{1 + \frac{1}{2}m} = 1$ or $\frac{m - \frac{1}{2}}{1 + \frac{1}{2}m} = -1$

Therefore, $m = 3$ or $m = -\frac{1}{3}$

Hence, slope of the other line is 3 or $-\frac{1}{3}$. Figure explains the reason of two answers.

S27. (i) The slope of the line through (3, –2) and (–1, 4) is

$$m = \frac{4-(-2)}{-1-3} = \frac{6}{-4} = -\frac{3}{2}.$$

(ii) The slope of the line through the points (3, –2) and (7, –2) is

$$m = \frac{-2-(-2)}{7-3} = \frac{0}{4} = 0.$$

(iii) The slope of the line through the points (3, –2) and (3, 4) is

$$m = \frac{4-(-2)}{3-3} = \frac{6}{0}.$$

which is not defined.

(iv) Here, inclination of the line $\alpha = 60°$. Therefore, slope of the line is

$$m = \tan 60° = \sqrt{3}.$$

S28. Slope of the line through the points (–2, 6) and (4, 8) is

$$m_1 = \frac{8-6}{4-(-2)} = \frac{2}{6} = \frac{1}{3}$$

Slope of the line through the points (8, 12) and (x, 24) is

$$m_2 = \frac{24-12}{x-8} = \frac{12}{x-8}$$

Since, two lines are perpendicular, $m_1 m_2 = -1$, which gives

$$\frac{1}{3} \times \frac{12}{x-8} = -1 \quad \text{or} \quad x = 4.$$

S29. (i) Here, slope of the line is $m = \tan\theta = \frac{1}{2}$ and y-intercept $c = -\frac{3}{2}$.

Therefore, by slope-intercept form the equation of the line is

$$y = \frac{1}{2}x - \frac{3}{2} \quad \text{or} \quad 2y - x + 3 = 0$$

which is the required equation.

(ii) Here, we have $m = \tan\theta = \frac{1}{2}$ and $d = 4$.

Therefore, by slope-intercept form the equation of the line is

$$y = \frac{1}{2}(x-4) \quad \text{or} \quad 2y - x + 4 = 0$$

which is the required equation.

S30. Given line is $x - 2y + 3 = 0$

can be written as $y = \frac{1}{2}x + \frac{3}{2}$ ... (i)

Slope of the line in Eq. (i) is $m_1 = \frac{1}{2}$.

Therefore, slope of the line perpendicular to line in Eq. (i) is

$$m_2 = -\frac{1}{m_1} = -2$$

Equation of the line with slope -2 and passing through the point $(1, -2)$ is

$$y - (-2) = -2(x-1) \quad \text{or} \quad y = -2x$$

which is the required equation.

S31. Given lines can be written as $y = -\frac{a_1}{b_1}x - \frac{c_1}{b_1}$ ... (i)

and $y = -\frac{a_2}{b_2}x - \frac{c_2}{b_2}$ ... (ii)

Slope of the lines Eq. (i) and (ii) are $m_1 = -\frac{a_1}{b_1}$ and $m_2 = -\frac{a_2}{b_2}$, respectively. Now,

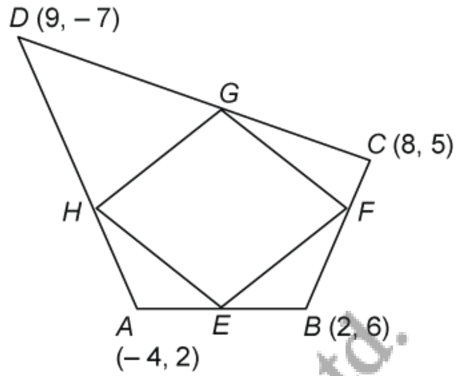
(i) Lines are parallel, if $m_1 = m_2$, which gives

$$-\frac{a_1}{b_1} = -\frac{a_2}{b_2} \quad \text{or} \quad \frac{a_1}{b_1} = \frac{a_2}{b_2}$$

(ii) Lines are perpendicular, if $m_1 \cdot m_2 = -1$, which gives

$$\left(-\frac{a_1}{b_1}\right)\cdot\left(-\frac{a_2}{b_2}\right) = -1 \quad \text{or} \quad a_1 a_2 + b_1 b_2 = 0.$$

S32. Let $A(-4, 2)$, $B(2, 6)$, $C(8, 5)$ and $D(9, -7)$ be the vertices of the quadrilateral $ABCD$. Let E, F, G, H be the mid points of AB, BC, CD and DA respectively.



Co-ordinates of E are $\left(\frac{-4+2}{2}, \frac{2+6}{2}\right) = (-1, 4)$

Co-ordinates of F are $\left(\frac{2+8}{2}, \frac{6+5}{2}\right) = \left(5, \frac{11}{2}\right)$

Co-ordinates of G are $\left(\frac{8+9}{2}, \frac{5-7}{2}\right) = \left(\frac{17}{2}, -1\right)$

Co-ordinates of H are $\left(\frac{9-4}{2}, \frac{-7+2}{2}\right) = \left(\frac{5}{2}, -\frac{5}{2}\right)$

$$\text{Slope of } EF = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\frac{11}{2} - 4}{5 - (-1)} = \frac{1}{4}$$

$$\text{Slope of GH} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-\frac{5}{2} - (-1)}{\frac{5}{2} - \frac{17}{2}} = \frac{1}{4}$$

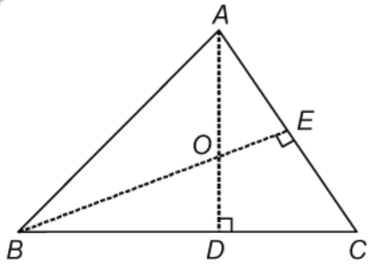
$\therefore$ Slope of EF = slope of GH $\Rightarrow$ $EF \parallel GH$.

Similarly, Slope of FG = slope of EH $\Rightarrow$ $FG \parallel EH$.

Since, $EF \parallel GH$ and $FG \parallel EH$, therefore $EFGH$ is a parallelogram.

S33. Let, A (10, 4), B (– 2, – 1) and C (– 4, 9).

We will get the coordinates of the orthocentre by solving the equation of the altitudes. (*i.e.*, AD and BE)



$$\text{Slope of } BC = \frac{9+1}{-4+2} = -5$$

$$\text{Therefore the slope of } AD = \frac{1}{5}$$

The equation of AD is $y - 4 = \frac{1}{5}(x - 10)$

$\Rightarrow$ $x - 5y + 10 = 0$ ... (i)

Now, the slope of $AC = \frac{9-4}{-4-10} = -\frac{5}{14}$

Therefore, the slope of $BE = \frac{14}{5}$

The equation BE is $y + 1 = \frac{14}{5}(x + 2)$ ... (ii)

Solving (i) and (ii), we get,

$$x = -1,\ y = \frac{9}{5}$$

Hence the orthocentre is $O\left(-1, \frac{9}{5}\right)$.

Date: 12/10/2021

Q1. Find the equation of the line which satisfy the given conditions: (i) Write the equations for the x- and y-axis, (ii) passing through the points (1, – 1) and (3, 5).

Q2. Find the equation of the line which satisfy the given conditions: Passing through the point (– 4, 3) with slope $\frac{1}{2}$.

Q3. Find the equation of the line, which passing through the points (– 1, 1) and (2, – 4).

Q4. Find the equation of the straight line which bisects the join of (5, 4) and (– 7, 0) and also bisects the join of (6, – 5) and (0, – 3).

Q5. Show that the points (1, 4), (3, – 2) and (– 3, 16) are collinear and find the equation of the line passing through these points.

Q6. The vertices of ΔPQR are P (2, 1), Q (– 2, 3) and R (4, 5). Find equation of the median through the vertex R.

Q7. Find the ratio in which the line joining (2, 3) and (4, 1) divides the line joining (1, 2) and (4, 3).

Q8. Find the equation of the line which is perpendicular to the line $\frac{x}{a} - \frac{y}{b} = 1$ at the point, where it meets the x-axis.

Q9. Show that the points $(at_1^2, 2at_1)$, $(at_2^2, 2at_2)$ and $(a, 0)$ are collinear if $t_1 t_2 = -1$.

Q10. Find the equation of the line which satisfy the given conditions: Passing through the point (– 1, 1) and (2, – 4).

Date: 12/10/2021

S1. (i) On x-axis, $y = 0$

Thus, equation of x-axis is $y = 0$.

On y-axis, $x = 0$

Thus, equation of y-axis is $x = 0$.

(ii) Here, $x_1 = 1$, $y_1 = -1$, $x_2 = 3$ and $y_2 = 5$. Using two-point form the equation of the line, we have

$$y - (-1) = \frac{5-(-1)}{3-1}(x-1)$$

$$\Rightarrow \quad y + 1 = \frac{5+1}{2}(x-1)$$

$$\Rightarrow \quad y + 1 = 3(x-1),$$

which is the required equation.

S2. The given line passes through $(-4, 3)$ and its slope $m = \frac{1}{2}$.

Now, equation of the line in point-slope form is:

$$y - y_1 = m(x - x_1)$$

or $$y - 3 = \frac{1}{2}(x+4)$$

or $$2(y-3) = x + 4$$

or $$2y - 6 = x + 4$$

or $$2y = x + 10$$

or $$x - 2y + 10 = 0.$$

S3. Here, $(x_1, y_1) = (-1, 1)$

and $(x_2, y_2) = (2, -4)$

We know that the equation of a line passing through two points (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\therefore \quad y - 1 = \frac{-4-1}{2+1}(x+1) \quad \Rightarrow \quad y - 1 = \frac{-5}{3}(x+1)$$

$$\Rightarrow \quad 3y - 3 = -5x - 5 \quad \Rightarrow \quad 5x + 3y + 2 = 0$$

S4. Mid-point of join of (5, 4) and (– 7, 0) is $\left(\frac{5-7}{2}, \frac{4+0}{2}\right)$ = (–1, 2)

Mid-point of join of (6, – 5) and (0, – 3) is $\left(\frac{6-0}{2}, \frac{-5-3}{2}\right)$ = (3, –4)

The equation of the line passing through the points (– 1, 2) and (3, – 4) is

$$y + 4 = \frac{2+4}{-1-3}(x-3), \quad y + 4 = -\frac{3}{2}(x-3)$$

$\Rightarrow \quad -2y - 8 = 3x - 9 \quad \Rightarrow \quad 3x + 2y = 1.$

S5. Equation of a line through (1, 4) and (3, – 2)

$$y - 4 = \frac{-2-4}{3-1}(x-1) \quad \Rightarrow \quad y - 4 = -3(x-1)$$

$\Rightarrow \quad y - 4 = -3x + 3 \quad \Rightarrow \quad 3x + y - 7 = 0$ ... (i)

Points will be collinear if (– 3, 16) lie on (i).

Now putting $x = -3$ and y = 16 in (i), we get 3 (– 3) + 16 – 7 = 0

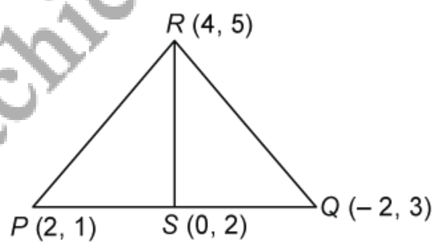
$\Rightarrow \quad -9 + 16 - 7 = 0 \quad \Rightarrow \quad 0 = 0$, which is true.

Hence, the points are collinear and the required equation of the line is $3x + y - 7 = 0$.

S6. Here, we have ΔPQR whose vertices are P (2, 1), Q (– 2, 3) and R (4, 5).

Let S be the mid-point of the side PQ.

The coordinates of S are $\left(\frac{2-2}{2}, \frac{1+3}{2}\right)$ *i.e.*, (0, 2).



RS is the median passing through R (4, 5) and S (0, 2).

The equation of RS is

$$y - 5 = \frac{2-5}{0-4}(x-4)$$

$\Rightarrow \quad y - 5 = \frac{3}{4}(x-4)$

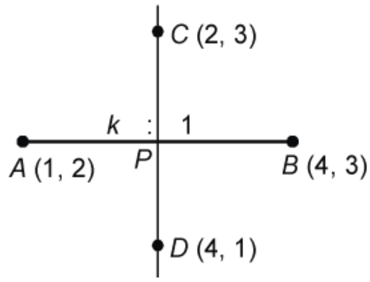
$\Rightarrow \quad 4y - 20 = 3x - 12$

$\Rightarrow \quad 3x - 4y + 8 = 0$

Hence, the equation of the median through R is $3x - 4y + 8 = 0$.

S7. Let the given points be A (1, 2), B (4, 3), C (2, 3) and D (4, 1).

Let CD divides AB at P in the ratio $k : 1$ internally.



$\therefore$ Coordinates of P are $\left(\frac{4k+1}{k+1}, \frac{3k+2}{k+1}\right)$

The equation of CD is

$$y - 3 = \frac{1-3}{4-2}(x-2) \qquad \left[\text{Using } y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)\right]$$

$$\Rightarrow \quad y - 3 = \frac{-2}{2}(x-2) \quad \Rightarrow \quad y - 3 = -x + 2$$

$$\Rightarrow \quad x + y - 5 = 0 \qquad \text{... (i)}$$

Since, P lies on CD, therefore the coordinates of P will satisfy the equation (i).

$$\therefore \quad \left(\frac{4k+1}{k+1}\right) + \left(\frac{3k+2}{k+1}\right) - 5 = 0 \quad \Rightarrow \quad 4k + 1 + 3k + 2 - 5k - 5 = 0$$

$$\Rightarrow \quad 2k - 2 = 0 \quad \Rightarrow \quad k = 1$$

Hence, the required ratio is 1 : 1.

S8. Here, we have $\quad \frac{x}{a} - \frac{y}{b} = 1 \qquad$ ... (i)

(i) meets the x-axis ($y = 0$) $\Rightarrow \frac{x}{a} + 0 = 1 \Rightarrow x = a$

Point of intersection of (i) and x-axis is (a, 0)

$$\text{Slope of (i)} = \frac{b}{a}$$

(i) is $\perp$ to the required line $\Rightarrow m_1 m_2 = -1$, $\frac{b}{a} m_2 = -1 \Rightarrow m_2 = -\frac{a}{b}$

Equation of the line passing through (a, 0) with slope $-\frac{a}{b}$ is

$$y - 0 = -\frac{a}{b}(x - a) \qquad [y - y_1 = m(x - x_1)]$$

$$\Rightarrow \quad by = -ax + a^2 \quad \Rightarrow \quad ax + by = a^2$$

S9. Given points are $(at_1^2, 2at_1)$, $(at_2^2, 2at_2)$ and $(a, 0)$.

The equation of the straight line passing through $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ is

$$y - 2at_1 = \frac{2at_2 - 2at_1}{at_2^2 - at_1^2}(x - at_1^2)$$

$$\Rightarrow \quad y - 2at_1 = \frac{2a(t_2 - t_1)}{a(t_2^2 - t_1^2)}(x - at_1^2) \qquad \left[\text{Using } y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)\right]$$

$$\Rightarrow \quad y - 2at_1 = \frac{2}{t_2 + t_1}(x - at_1^2)$$

$$\Rightarrow \quad (t_1 + t_2)(y - 2at_1) = 2(x - at_1^2) \qquad \dots \text{(i)}$$

Given three points are collinear if (a, 0) lies on (i).

$$\Rightarrow \quad \text{if } (t_1 + t_2)(0 - 2at_1) = 2(a - at_1^2) \quad \Rightarrow \quad \text{if} \quad -2at_1^2 - 2at_1t_2 = 2a - 2at_1^2$$

$$\Rightarrow \quad \text{if} \quad -2at_1t_2 = 2a \quad \Rightarrow \quad \text{if} \quad t_1t_2 = -1$$

Hence, the given points are collinear if $t_1t_2 = -1$.

S10. The line passes through (– 1, 1) and (2, –4).

Now, Slope of the line = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 1}{2 + 1} = \frac{-5}{3}$

Applying two-point form of the equation of the line, we get

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 1 = \frac{-5}{3}(x + 1)$$

$$3y - 3 = -5x - 5$$

$$5x + 3y + 2 = 0.$$

Date: 12/10/2021

Q1. Reduce the equation 6x + 3y – 5 = 0 to the slope-intercept form and find its slope and *y*-intercept.

Q2. Find the equation of the line which makes intercepts 2 and – 3 on the *x*-axis and the *y*-axis respectively.

Q3. Find the intercepts cut off by the line 2x – y + 16 = 0 on the coordinate axes.

Q4. Find the equation of the straight line passing through (2, 3) and cutting off equal intercepts along the positive directions of both the axes.

Q5. A straight line passes through the point (2, 3) and the portion of the line intercepted between the axes is bisected at this point. Find the equation of the line.

Q6. Find the equation of a line whose slope is $\frac{1}{2}$ and *y*-intercept equal to $\frac{-5}{4}$.

Q7. Find the equation of a line which intersects the y-axis at a distance of 3 units above the origin and makes an angle of 30° with the positive direction of the *x*-axis.

Q8. Find the equation of a line which cuts off an intercept of a 4 units on negative direction of the *y*-axis and makes an angle of 120° with the positive direction of the *x*-axis.

Q9. Find the equation of a line which passes through the point (– 3, 7) and makes intercepts on the axes, equal in magnitude but opposite in sign.

Q10. Find the area of the triangle formed by the coordinate axes and the line $ax + by = 2ab$.

Q11. Find the equations of the lines, which cut-off intercepts on the axes whose sum and product are 1 and – 6 respectively.

Q12. Find the equation of the straight line which passes through the point (3, – 2) and cuts off positive intercepts on the *x*-axis and *y*-axis which are in the ratio 4 : 3.

Q13. Find the equation of the line passing through the point (1, 3) such that the intercept on the *y*-axis exceeds the intercepts on the *x*-axis by 4.

Q14. Find the equation of a line drawn perpendicular to the line $\frac{x}{4} + \frac{y}{6} = 1$ through the point, where it meets the *y*-axis.

Q15. If *M*(*a*, *b*) is the midpoint of a line segment intercepted between the the axes, show that the equation of the line is $\frac{x}{a} + \frac{y}{b} = 2$.

Q16. Find the equation of the line which passes through the point (3, 4) and the sum of whose intercepts on the axis is 14.

Q17. Write down the equation of the line which makes an intercept of 2*a* on the *x*-axis and 3*a* on the *y*-axis. Given that the line passes through the point (14, – 9), find the numerical value of *a*.

Q18. If *P* be the measure of the perpendicular segment from the origin on the line whose intercepts on the axes are *a* and *b*, show that:

$$\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}$$

Q19. If the point *R* (*h*, *k*) divides a line segment between the axis in the ratio 1 : 2. Find the equation of the line.

Q20. Find the equations of the lines passing through the point (2, 2), such that sum of their intercepts on the axis is 9.

Date: 12/10/2021

S1. We know that the equation of a line with slope m and y-intercept c is given by $y = mx + c$.

Now, $6x + 3y - 5 \Rightarrow 3y = -6x + 5$

$$\Rightarrow \quad y = -2x + \frac{5}{3}$$

$\therefore \quad m = -2$ and $c = \frac{5}{3}$.

Hence, slope = – 2 and y-intercept = $\frac{5}{3}$.

S2. We know that the equation of a line making intercepts a and b on the x-axis and y-axis respectively, is $\frac{x}{a} + \frac{y}{b} = 1$.

Here, $a = 2$ and $b = -3$

Hence, the required equation is

$$\frac{x}{2} + \frac{y}{-3} = 1 \quad \Leftrightarrow \quad \frac{x}{2} - \frac{y}{3} = 1 \quad \Leftrightarrow \quad 3x - 2y - 6 = 0.$$

S3. We have

$$2x - y + 16 = 0 \quad \Leftrightarrow \quad 2x - y = -16 \quad \Leftrightarrow \quad \frac{x}{-8} + \frac{y}{16} = 1$$

[on dividing both sides by – 16]

$\therefore$ the x-intercept = – 8 and the y-intercept = 16.

S4. Let, $a = k$, $b = k$

So, the equation of the line is

$$\frac{x}{k} + \frac{y}{k} = 1 \qquad \left[\text{Using } \frac{x}{a} + \frac{y}{b} = 1\right]$$

$$\Rightarrow \quad x + y = k \qquad \text{... (i)}$$

Since, the line (i) passes through (2, 3).

$\therefore \quad 2 + 3 = k \quad \Rightarrow \quad k = 5$

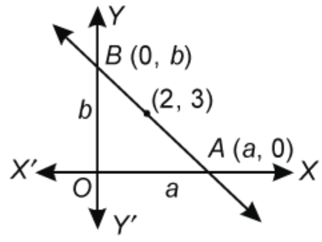
Substituting $k = 5$ in (i), we get

$$x + y = 5$$

which is required equation of the line.

S5. Let the intercepts of the line be a and b and the line intersects the x-axis and y-axis at A and B.

$\therefore$ $A(a, 0)$ and $B(0, b)$ are on the line and the mid-point of AB is $(2, 3)$.



$$\therefore \qquad 2 = \frac{a+0}{2} \Rightarrow a = 4$$

and $$3 = \frac{0+b}{2} \Rightarrow b = 6$$

Thus, the equation of the line in the intercept form is

$$\frac{x}{4} + \frac{y}{6} = 1 \qquad \left[\text{Using } \frac{x}{a} + \frac{y}{b} = 1\right]$$

$$\Rightarrow \qquad 3x + 2y - 12 = 0$$

S6. We know that the equation of a line with slope m and y-intercept c is given by $y = mx + c$.

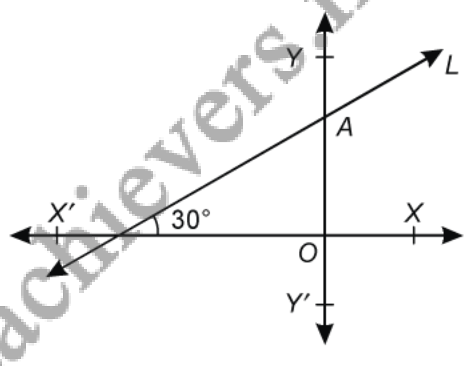
Here, $$m = \frac{1}{2} \quad \text{and} \quad c = \frac{-5}{4}$$

Hence, the required equation of the line is

$$y = \frac{1}{2}x - \frac{5}{4} \Rightarrow 4y = 2x - 5$$

$$\Rightarrow 2x - 4y - 5 = 0$$

S7. We know that the equation of a line with slope m and y-intercept c is given by $y = mx + c$.



Here, $$m = 30° = \frac{1}{\sqrt{3}} \text{ and } c = 3$$

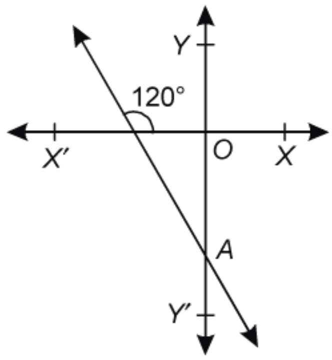
Hence, the required equation of the line is

$$y = \frac{1}{\sqrt{3}}x + 3 \Rightarrow \sqrt{3}\,y = x + 3\sqrt{3} \qquad [\because \; y = mx + c]$$

$$\Rightarrow x - \sqrt{3}\,y + 3\sqrt{3} = 0$$

S8. We know that the equation of a line with slope m and y-intercept c is given by

$$y = mx + c$$



Here, $$m = \tan 120° = \tan(180° - 60°)$$

$$= -\tan 60° = -\sqrt{3} \quad \text{and} \quad c = -4.$$

Hence, the required equation of the line is

$$y = -\sqrt{3}\,x - 4$$

$$\Rightarrow \qquad \sqrt{3}\,x + y + 4 = 0 \qquad [\because \; y = mx + c]$$

S9. Let the required line make intercepts a and $-a$ on the x-axis and y-axis respectively.

Then, its equation is $\frac{x}{a} + \frac{y}{-a} = 1 \Leftrightarrow x - y = a$

Since this line passes through the point (– 3, 7), we have

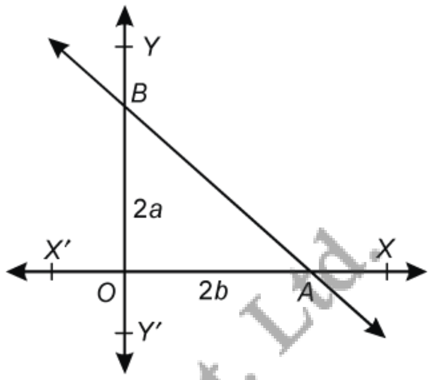
$$a = (-3 - 7) = -10$$

So, the required equation of the line is

$$\frac{x}{-10} + \frac{y}{10} = 1 \;\Leftrightarrow\; -x + y = 10 \;\Leftrightarrow\; x - y + 10 = 0$$

Hence, the required equation is $x - y + 10 = 0$.

S10. Let $X'OX$ and YOY' be the coordinate axes.



Now, $$ax + by = 2ab \;\Rightarrow\; \frac{x}{2b} + \frac{y}{2a} = 1$$

Suppose that this line meets the x-axis and y-axis at A and B respectively.

Then, $$OA = 2b \quad \text{and} \quad OB = 2a.$$

$\therefore$ $$\text{area of } \Delta BOA = \left(\frac{1}{2} \times OA \times OB\right) \text{ sq units}$$

$$= \left(\frac{1}{2} \times 2b \times 2a\right) = 2ab \text{ sq units}$$

S11. Let the intercepts be a and b.

Thus, $$a + b = 1, \quad ab = -6$$

Now, $$(a - b)^2 = (a + b)^2 - 4ab$$
$$= (1)^2 - 4(-6)$$
$$= 1 + 24 = 25$$

or $$a - b = \pm 5$$

Case (i) : When $a - b = 5$ and $a + b = 1$

We have, $a = 3, \quad b = -2$

Equation of the line is

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\Rightarrow \quad \frac{x}{3} + \frac{y}{-2} = 1$$

$$\Rightarrow \quad 2x - 3y = 6$$

Case (ii) : When $a - b = -5$ and $a + b = 1$

We have, $a = -2, \quad b = 3$

Equation of the line is

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\Rightarrow \quad \frac{x}{-2} + \frac{y}{3} = 1$$

$$\Rightarrow \quad -3x + 2y = 6$$

$$\Rightarrow \quad 3x - 2y = -6$$

Hence, required equations of the lines are

$$2x - 3y = 6 \quad \text{and} \quad 3x - 2y = -6.$$

S12. The equation of the line in intercept form is given by

$$\frac{x}{a} + \frac{y}{b} = 1 \qquad \text{... (i)}$$

Where, x-intercept = a, y-intercept = b

Given intercept on x-axis and y-axis are in the ratio 4 : 3

So, Let take $a = 4k, \quad b = 3k$

$$\frac{x}{4k} + \frac{y}{3k} = 1 \qquad \text{... (ii)}$$

Also, line passes through the point (3, – 2), we get

$$\frac{x}{4k} + \frac{y}{3k} = 1 \qquad \text{[Point (3, – 2) satisfies the equation of line (ii)]}$$

$$\frac{9-8}{12k} = 1 \Rightarrow 1 = 12k \Rightarrow k = \frac{1}{12}$$

So x-intercept $= a = 4k = \frac{4}{12} = \frac{1}{3}$

y-intercept $= b = 3k = \frac{3}{12} = \frac{1}{4}$

Thus, from (i), we get the equation of the line $\frac{x}{\frac{1}{3}} + \frac{y}{\frac{1}{4}} \Rightarrow 3x + 4y = 1$

Hence the required equation of the line is $3x + 4y = 1$.

S13. According to the given condition $b = a + 4$

The equation of the line will be $\frac{x}{a} + \frac{y}{a+4} = 1$

As it passes through the point (1, 3), we get

$$\frac{1}{a}+\frac{3}{a+4} = 1$$

$\Rightarrow$ $a + 4 + 3a = a\,(a + 4)$

$\Rightarrow$ $a^2 = 4 \quad \Rightarrow \quad a = \pm 2$

When $a = 2$, $b = 6$, and when $a = -2$, $b = 2$

The equation will be $\frac{x}{2}+\frac{y}{6} = 1$ and $\frac{x}{-2}+\frac{y}{2} = 1$

$\Rightarrow$ $3x + y = 6$ and $y - x = 2$.

S14. The given line is $\frac{x}{4}+\frac{y}{6} = 1$

It meets the y-axis at $B\,(0, 6)$.

Also, $\frac{x}{4}+\frac{y}{6} = 1 \Rightarrow 3x + 2y = 12$

$$\Leftrightarrow y = \frac{-3}{2}x+6$$

Slope of the given lines is $\frac{-3}{2}$.

Let the slope of the line perpendicular to the given line be m. Then

$$m \times \left(\frac{-3}{2}\right) = -1 \quad \Rightarrow \quad m = \frac{2}{3} \qquad [\because \quad m_1 m_2 = -1]$$

Now, the required line has slope $\frac{2}{3}$ and passes through the point $B(0, 6)$. So, its equation is

$$\frac{y-6}{x-0} = \frac{2}{3} \quad \Leftrightarrow \; 2x = 3y - 18 \; \Leftrightarrow \; 2x - 3y + 18 = 0$$

Hence, the required equation is $2x - 3y + 18 = 0$.

S15. Let the required equation of the line be

$$\frac{x}{c}+\frac{y}{d} = 1$$

Then, it cuts the x-axis and y-axis at the points $A(c, 0)$ and $B(0, d\,)$.

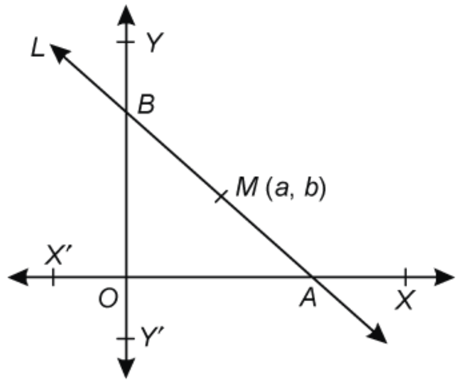
Then, $\frac{c+0}{2} = a$ and $\frac{0+d}{2} = b$

$\Leftrightarrow$ $c = 2a$ and $d = 2b$

So, the required equation is

$$\frac{x}{2a} + \frac{y}{2b} = 1 \quad \Leftrightarrow \quad \frac{x}{a} + \frac{y}{b} = 2$$

Hence, the required equation is $\frac{x}{a} + \frac{y}{b} = 2$.



S16. Let the intercepts made by the line on the x-axis and y-axis be a and $(14 - a)$ respectively.

Then, its equation is $\frac{x}{a} + \frac{y}{(14-a)} = 1$ [intercept form]

Since it passes through the point (3, 4), we have

$$\frac{3}{a} + \frac{4}{(14-a)} = 1 \Leftrightarrow 3(14-a) + 4a = a(14-a)$$

$$\Leftrightarrow a^2 - 13a + 42 = 0 \quad \Leftrightarrow \quad (a-6)(a-7) = 0$$

$$\Leftrightarrow a = 6 \quad \text{or} \quad a = 7$$

Now, $a = 6, \quad b = 14 - 6 = 8$

And, $a = 7, \quad b = 14 - 7 = 7$

So, the required equation is

$$\frac{x}{6} + \frac{y}{8} = 1 \quad \text{or} \quad \frac{x}{7} + \frac{y}{7} = 1$$

i.e., $4x + 3y - 24 = 0$ or $x + y - 7 = 0$.

S17. The equation of the line in intercept form is given by $\frac{x}{a} + \frac{y}{b} = 1$ ... (i)

where x-intercept $= a$, y-intercept $= b$

According to the question given x-intercept $= 2a$, y-intercept $= 3a$

So $\boxed{b = 3a}$

From (i), we get $\frac{x}{2a} + \frac{y}{3a} = 1$ ... (ii)

Also line passes through the point (14, – 9)

So, point $P(14, -9)$ satisfied the equation (ii), we get $\frac{14}{2a} - \frac{9}{3a} = 1$

$$\frac{14-18}{6a} = 1 \quad \Rightarrow \quad 6a = 24 \quad \Rightarrow \quad \boxed{a = 4}$$

So, x-intercept = 8 and y-intercept = 12

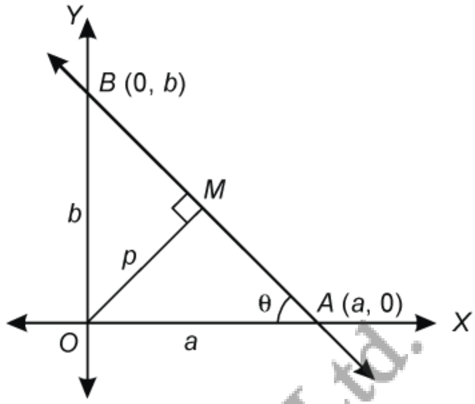
Thus, the equation of the line becomes $\frac{x}{8} + \frac{y}{12} = 1 \Rightarrow \frac{3x + 2y}{24} = 1$

Hence, the required equation of the line is $3x + 2y = 24$

S18. Let the line meets the x-axis in A and y-axis in B, then $OA = a$ and $OB = b$.

Draw $OM \perp AB$ then $OM = p$

Let $\angle OAM = \theta$, the $\angle MOA = 90° - \theta$, $\angle BOM = \theta$.



From right angled triangle OMA,

$$\sin\theta = \frac{p}{a} \qquad \text{... (i)}$$

From right angled triangle OMB,

$$\cos\theta = \frac{p}{b} \qquad \text{... (ii)}$$

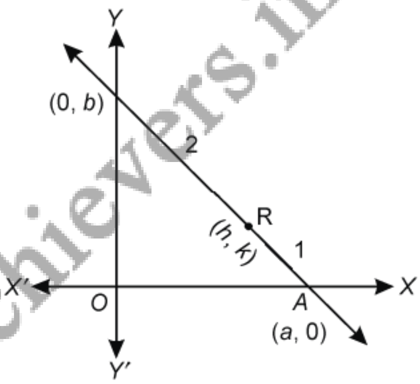
Squaring (i) and (ii), and adding, we get

$$\sin^2\theta + \cos^2\theta = \frac{p^2}{a^2} + \frac{p^2}{b^2} \Rightarrow 1 = p^2\left(\frac{1}{a^2} + \frac{1}{b^2}\right)$$

$$\Rightarrow \qquad \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2}.$$

S19. Let a line AB intersect x-axis at $(a, 0)$ and y-axis at $(0, b)$

By section formula



$$h = \frac{1\times 0 + 2a}{1+2} = \frac{2a}{3} \Rightarrow a = \frac{3h}{2}$$

$$k = \frac{1\times b + 2\times 0}{3} = \frac{b}{3} \Rightarrow b = 3k$$

Equation of the line with intercepts a and b is

$$\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow \frac{x}{\frac{3h}{2}} + \frac{y}{3k} = 1 \Rightarrow \frac{2x}{3h} + \frac{y}{3k} = 1$$

$$\Rightarrow \qquad \frac{2x}{h} + \frac{y}{k} = 3 \Rightarrow 2kx + hy = 3kh$$

S20. The equation of a line in intercept form is given by

$$\frac{x}{a} + \frac{y}{b} = 1 \qquad \text{... (i)}$$

Where, x-intercept = a, y-intercept = b

given $a + b = 9 \Rightarrow b = 9 - a$

So, line (i) becomes $\frac{x}{a} + \frac{y}{9-a} = 1$ ... (ii)

Also line passes through the point (2, 2)

So, $P(2, 2)$ satisfies the line (ii), we get by putting $x = 2$, $y = 2$

$$\frac{2}{a} + \frac{2}{9-a} = 1 \Rightarrow \frac{2(9-a)+2a}{a(9-a)} = 1$$

$$\Rightarrow 18 - 2a + 2a = 9a - a^2 \Rightarrow a^2 - 9a + 18 = 0$$

$$\Rightarrow a^2 - 6a - 3a + 18 = 0 \Rightarrow a(a-6) - 3(a-6) = 0$$

$$\Rightarrow (a-6)(a-3) = 0 \Rightarrow a = 6,\ a = 3$$

(i) Put $a = 6$, we get $\frac{x}{6} + \frac{y}{3} = 1 \Rightarrow 3x + 6y = 18 \Rightarrow x + 2y = 6$

(ii) Put $a = 3$, we get $\frac{x}{3} + \frac{y}{6} = 1 \Rightarrow 6x + 3y = 18 \Rightarrow 2x + y = 6$

Hence, the required equations of lines are $x + 2y = 6$ or $2x + y = 6$.

Date: 12/10/2021

Q1. Write down the equation of the line for which : $p = 3$ and $\alpha = 120°$.

Q2. The perpendicular distance of a line from the origin is 7 cm and its slope is – 1. Find the equation of the line.

Q3. Find the equation of a line whose perpendicular distance from the origin is 5 units and the angle between the positive direction of the x-axis and the perpendicular is 30°.

Q4. Find the Equation of line if perpendicular distance from the origin is 5 units and the angle made by the perpendicular with the positive x-axis is 30°.

Q5. Reduce the equation $\sqrt{3}\,x + y - 8 = 0$ into normal form. Find the values of p and ω.

Q6. Find the equation of the line whose perpendicular distance from the origin is 3 units and the angle between the positive direction of x-axis and the perpendicular is 15°.

Q7. Find the equation of a line whose perpendicular distance from the origin is $\sqrt{8}$ units and the angle between the positive direction of the x-axis and the perpendicular is 135°.

Q8. Find the equation of a line whose perpendicular distance from the origin is 2 units and the angle between the perpendicular segment and the positive direction of the x-axis is 240°.

Q9. Find the equations of two straight line which are at a distance $\frac{1}{2}$ unit from the origin and pass through the point (0, 1).

Q10. Find the equation of straight line for which $p = 5$, $\alpha = 135°$.

Q11. Find the values of θ and p, if the equation $x \cos\theta + y \sin\theta = p$ is the normal form of the line $\sqrt{3}\,x + y + 2 = 0$.

Q12. Find the equation of the line whose perpendicular distance from the origin is 4 units and the angle which the normal makes with positive direction of x-axis is 15°.

Q13. A straight canal is $4\frac{1}{2}$ miles from a place and the shortest route from this place to the canal is exactly north-east. A village is 3 miles north and four miles east from the place. Does it lie by the nearest edge of the canal?

Q14. Find the equation of the line for which $p = 2$, $\sin\alpha = \frac{4}{5}$.

Q15. Find the equation of the straight line at a distance of 3 units from the origin such that the perpendicular from the origin to the line makes an angle α, given by $\tan\alpha = \frac{5}{12}$, with the positive direction of x-axis.

Q16. Find the equation of two straight lines which are at a distance of 3 units from the origin and passing through the point (6, 0).

Date: 12/10/2021

S1. Equation of line in perpendicular form is given by $x \cos \alpha + y \sin \beta = p$

Here, $p = 3$ and $\alpha = 120°$

The equation of the required line is

$$x \cos 120° + y \sin 120° = 3$$

$$\Rightarrow \quad x\left(-\frac{1}{2}\right) + y\left(\frac{\sqrt{3}}{2}\right) = 3 \quad \Rightarrow \quad x - y\sqrt{3} + 6 = 0$$

S2. Equation of line in perpendicular form is given by $x \cos \alpha + y \sin \beta = p$

Here, $m = \tan \alpha = -1 \quad \Rightarrow \quad \alpha = \tan^{-1}(-1) = 135°$ and $p = 7$

Thus, the required line is

$$x \cos 135° + y \sin 135° = 7 \qquad [\text{Using } x \cos \alpha + y \sin \alpha = p]$$

$$\Rightarrow \quad x \cos (180° - 45°) + y \sin (180° - 45°) = 7$$

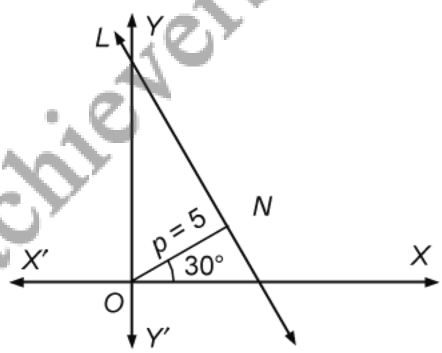
$$\Rightarrow \quad -x \cos 45° + y \sin 45° = 7$$

$$\Rightarrow \quad -x \cdot \frac{1}{\sqrt{2}} + y \cdot \frac{1}{\sqrt{2}} = 7 \quad \Rightarrow \quad x - y + 7\sqrt{2} = 0$$

S3. Here, $p = 5$ units and $\alpha = 30°$.

So, the equation of the given line in normal form is

$x \cos \alpha + y \sin \alpha = p$, where $\alpha = 30°$ and $p = 5$ units



$$\Leftrightarrow \quad x \cos 30° + y \sin 30° = 5$$

$$\Leftrightarrow \quad x\left(\frac{\sqrt{3}}{2}\right) + y\left(\frac{1}{2}\right) = 5$$

$$\Leftrightarrow \quad \sqrt{3}\,x + y - 10 = 0,$$ which is the required equation.

S4. Given : The perpendicular distance from the origin is 5 units, *i.e.*, $p = 5$ and $\omega = 30°$.

Now, equation of the line in normal form is

$$x \cos \omega + y \sin \omega = p$$

$$x \cos 30° + y \sin 30° = 5$$

$$x \times \frac{\sqrt{3}}{2} + y \times \frac{1}{2} = 5$$

$$\sqrt{3}x + y = 10 \quad \text{(Required equation.)}$$

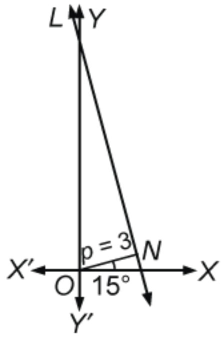
S5. Given $\sqrt{3}\,x + y - 8 = 0$ ... (i)

Dividing Eq. (i) by $\sqrt{(\sqrt{3})^2 + (1)^2} = 2$, we get

$$\frac{\sqrt{3}}{2}x + \frac{1}{2}y = 4 \quad \text{or} \quad x\cos 30° + y\sin 30° = 4 \qquad \text{... (ii)}$$

Comparing Eq. (ii) with $x\cos\omega + y\sin\omega = p$, we get $p = 4$ and $\omega = 30°$.

S6. Here, $p = 3$ units and $\alpha = 15°$.



So, the equation of the given line in normal form is

$$x\cos\alpha + y\sin\alpha = p,$$

where, $\alpha = 15°$ and $p = 3$ units

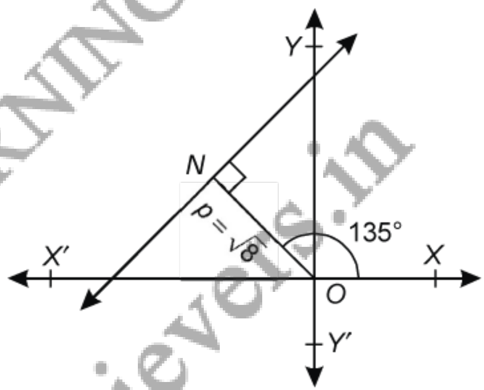
$$\Leftrightarrow \quad x\cos 15° + y\sin 15° = 3$$

$$\Leftrightarrow \quad \frac{x(\sqrt{3}+1)}{2\sqrt{2}} + \frac{y(\sqrt{3}-1)}{2\sqrt{2}} = 3$$

[$\because$ cos 15° = cos (45° – 30°) = cos 45° cos 30° + sin 45° sin 30° and sin 15° = sin (45° – 30°) = sin 45° cos 30° – cos 45° sin 30°]

$\Leftrightarrow (\sqrt{3} + 1)x + (\sqrt{3} - 1)y - 6\sqrt{2} = 0$, which is the required equation.

S7. Here, $p = \sqrt{8}$ units and $\alpha = 135°$



So, the equation of the given line in normal form is

$$x\cos\alpha + y\sin\alpha = p,$$

where, $\alpha = 135°$ and $p = \sqrt{8}$ units

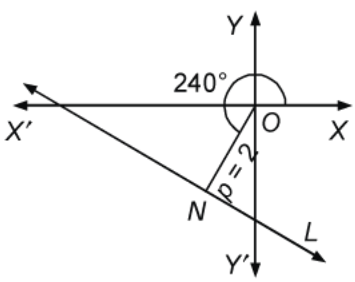
$$\Leftrightarrow \quad x\cos 135° + y\sin 135° = \sqrt{8}$$

$$x\left(\frac{-1}{\sqrt{2}}\right) + y\left(\frac{1}{\sqrt{2}}\right) = \sqrt{18}$$

[$\because$ cos 135° = cos (180° – 45°) = – cos 45° and sin 135° = sin (180° – 45°) = sin 45°]

$\Leftrightarrow \quad -x + y = 4 \quad \Leftrightarrow \quad x - y + 4 = 0$, which is the required equation.

S8. Here, $p = 2$ units and $\alpha = 240°$



So, the equation of the given line in normal form is

$$x\cos\alpha + y\sin\alpha = p,$$

where, $\alpha = 240°$ and $p = 2$ units

$$\Leftrightarrow \quad x\cos 240° + y\sin 240° = 2$$

$$\Leftrightarrow \quad x\left(\frac{-1}{\sqrt{2}}\right) + y\left(\frac{-\sqrt{3}}{\sqrt{2}}\right) = 2$$

[$\because$ cos 240° = cos (180° + 60°) = – cos 60° and sin 240° = sin (180° + 60°) = – sin 60°]

$\Leftrightarrow \quad -x - \sqrt{3}\,y = 4 \quad \Leftrightarrow \quad x + \sqrt{3}\,y + 4 = 0$, which is the required equation.

S9. The equation of any line which is at a distance $\frac{1}{2}$ from the origin is

$$x \cos \alpha + y \sin \alpha = \frac{1}{2} \qquad \text{... (i)}$$

It passes through the point (0, 1).

$$\therefore \quad (0) \cos \alpha + (1) \sin \alpha = \frac{1}{2} \quad \Rightarrow \quad \sin \alpha = \frac{1}{2}$$

$$\therefore \quad \cos \alpha = \pm\sqrt{1 - \sin^2 \alpha} = \pm \sqrt{1 - \left(\frac{1}{2}\right)^2} = \pm \sqrt{1 - \frac{1}{4}} = \pm \frac{\sqrt{3}}{2}$$

Putting these values of sin α and cos α in (i), we get

$$\pm \frac{\sqrt{3}}{2} x + \frac{1}{2} y = \frac{1}{2} \quad \Rightarrow \quad \pm \sqrt{3}\, x + y = 1$$

$$\Rightarrow \quad \sqrt{3}\, x + y - 1 = 0 \quad \text{and} \quad -\sqrt{3}\, x + y - 1 = 0$$

which are the required equations of the lines.

S10. Normal form of the equation of the straight line is given by $x \cos \alpha + y \sin \alpha = p$ in which p is the length of the perpendicular from the origin and α is the angle which this perpendicular makes with x-axis.

Here, p = perpendicular length = 5 units

$$\alpha = 135°$$

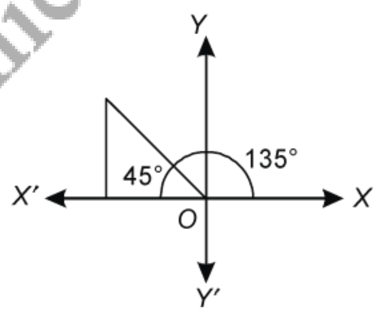
$$\cos \alpha = \cos 135° = \cos (180° - 45°) = - \cos 45°$$

because cos θ is negative in the second quadrant.

$$\cos \alpha = \cos 135° = -\frac{1}{\sqrt{2}}$$

$$\sin \alpha = \sin 135° = \sin (180° - 45°)$$

$$\sin \alpha = \sin 45° = \frac{1}{\sqrt{2}}$$



[∵ sin θ is positive in the second quadrant]

So from the equation of the straight line

$x \cos \alpha + y \sin \alpha = p$, we get

$$x\left(-\frac{1}{\sqrt{2}}\right) + y\left(\frac{1}{\sqrt{2}}\right) = 5 \quad \Rightarrow \quad -x + y = 5\sqrt{2} \quad \Rightarrow \quad x - y + 5\sqrt{2} = 0$$

Hence the required equation of the straight line is $x - y + 5\sqrt{2} = 0$.

S11. Given $\sqrt{3}\,x + y + 2 = 0$

$$-\sqrt{3}\,x - y = -2$$

Now, $a = -\sqrt{3}, \quad b = -1$

Now, $\sqrt{a^2 + b^2} = \sqrt{3+1} = 2$

$$-\frac{\sqrt{3}}{2}x - \frac{1}{2}y = \frac{2}{2}$$

$\Rightarrow$ $-\frac{\sqrt{3}}{2}x - \frac{1}{2}y = 1$

Comparing with $x \cos\theta + y \sin\theta = p$

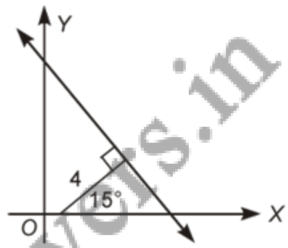
$$p = 1, \quad \cos\theta = -\frac{\sqrt{3}}{2}$$

and $\sin\theta = -\frac{1}{2}$

$\Rightarrow$ $\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

Hence, $p = 1, \quad \theta = \frac{7\pi}{6}$.

S12. Here, we are given $p = 4$ and $\omega = 15°$ (see figure)



Now, $\cos 15° = \frac{\sqrt{3}+1}{2\sqrt{2}}$

and $\sin 15° = \frac{\sqrt{3}+1}{2\sqrt{2}}$

By the normal form the equation of the line is

$$x \cos 15° + y \sin 15° = 4$$

$$x\frac{\sqrt{3}+1}{2\sqrt{2}} + y\frac{\sqrt{3}-1}{2\sqrt{2}} = 4$$

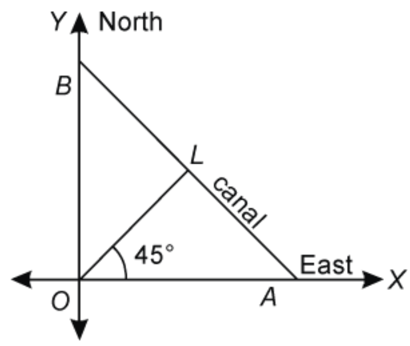
$$x(\sqrt{3}+1) + y(\sqrt{3}-1) = 8\sqrt{2}$$

This is the required equation.

S13. Let the given place be O. Take this as the origin and the east and north directions through O as the x and y axes respectively.

Let AB be the nearest edge of the canal. From the question OL is perpendicular to AB such that

$OL = 4\frac{1}{2}$ miles and $\angle LOA = 45°$.



So, the equation of the canal is

$$x \cos 45° + y \sin 45° = 4\frac{1}{2}$$

$$\Rightarrow \quad \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = \frac{9}{2}$$

$$\Rightarrow \quad \sqrt{2}\,(x + y) = 9 \qquad \text{... (i)}$$

The position of the village is (4, 3). The village will lie on the edge of the canal if (4, 3) satisfies the equation (i). Clearly (4, 3) does not satisfy (i). Hence the village does not lie by the nearer edge of the canal.

S14. The equation of the straight line on which the length of the perpendicular from the origin and the angle which this perpendicular makes with x-axis is given by

$$x \cos \alpha + y \sin \alpha = p$$

Here $\quad p = 2, \quad$ and $\sin \alpha = \frac{4}{5}$ ... (i)

$$\Rightarrow \quad \sin^2 \alpha = \frac{16}{25} \Rightarrow 1 - \cos^2 \alpha = \frac{16}{25} \Rightarrow \cos^2 \alpha = 1 - \frac{16}{25}$$

$$\Rightarrow \quad \cos^2 \alpha = \frac{25 - 16}{25} \Rightarrow \cos^2 \alpha = \frac{9}{25} \Rightarrow \cos \alpha = \pm\frac{3}{5}$$

Case-I: Take $\cos \alpha = \frac{3}{5}$

So the equation

$x \cos \alpha + y \sin \alpha = p$ becomes

$$x \times \frac{3}{5} + y \times \frac{4}{5} = 2 \Rightarrow 3x + 4y = 10 \Rightarrow 3x + 4y - 10 = 0$$

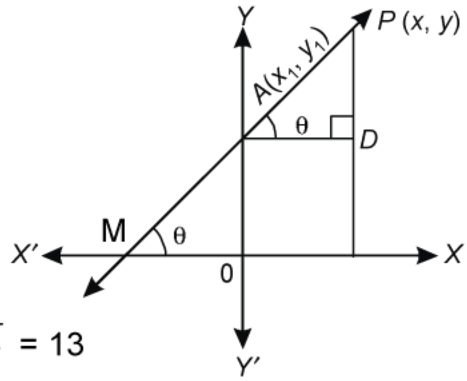
Case-II: Take $\cos \alpha = \frac{-3}{5}$ we get

$$x\left(\frac{-3}{5}\right) + y \times \frac{4}{5} = 2 \Rightarrow -3x + 4y = 10 \Rightarrow 3x - 4y + 10 = 0$$

Hence the required equation of the straight lines are

$$3x + 4y - 10 = 0, \quad 3x - 4y + 10 = 0.$$

S15. The equation of the straight line on which the length of the perpendicular from the origin and the angle which this perpendicular makes with x-axis given by

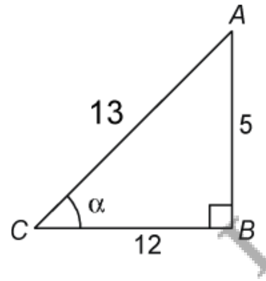


$$x \cos\alpha + y \sin\alpha = p$$

Here $\qquad p = 3$ and $\tan\alpha = \frac{5}{12}$

$\Rightarrow \qquad = \frac{\text{perpendicular}}{\text{Base}} = \frac{5}{12}$

$$AC = \sqrt{AB^2 + BC^2} = \sqrt{25+144} = 13$$



$$\sin\alpha = \frac{AB}{AC} = \frac{5}{13},$$

$$\cos\alpha = \frac{BC}{AC} = \frac{12}{13}$$

So, equation $\quad x\cos\alpha + y\sin\alpha = p$ becomes

$$x\frac{12}{13} + y\frac{5}{13} = 3 \Rightarrow 12x + 5y = 39 \Rightarrow 12x + 5y - 39 = 0$$

Also tan α is +ve in I and III quadrant. If a lies in the third quadrant then sin α and cos α take –ve value.

$$x\left(\frac{-12}{13}\right) + y\left(\frac{-5}{13}\right) = 3 \Rightarrow -12x - 5y = 39 \Rightarrow 12x + 5y + 39 = 0$$

Hence the required equation of the lines are

$$12x + 5y + 39 = 0$$
$$12x + 5y - 39 = 0.$$

S16. Let m be the slope of the line. The equation of the line in point-slope form is given by

$$y - y_1 = m(x - x_1) \qquad \text{... (i)}$$

Here, $\quad$ point $P(x_1, y_1)$ is (6, 0)

So take $\qquad x_1 = 6, \quad y = 0$, we get from (i)

$$y - 0 = m(x - 6) \Rightarrow y = mx - 6m \Rightarrow mx - y - 6m = 0 \qquad \text{... (ii)}$$

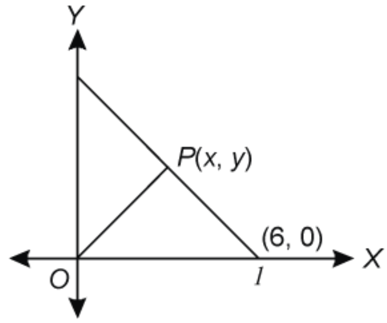
Also length of perpendicular from the origin is 3 units.

$$d = \left|\frac{mx - y - 6m}{\sqrt{m^2 + (-1)^2}}\right| \qquad \text{Using the formula } d = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}\right|$$

Line passes through (0, 0). So we get d = $\left|\frac{m \times 0 - 0 - 6m}{\sqrt{m^2+1}}\right| \Rightarrow \left|\frac{-6m}{\sqrt{m^2+1}}\right| = 3$

$$\Rightarrow \qquad \frac{6m}{\sqrt{m^2+1}} = 3 \Rightarrow 6m = 3(\sqrt{m^2+1})$$

Squaring both sides, we get



$$36m^2 = 9(m^2 + 1) \Rightarrow 36m^2 - 9m^2 = 9$$

$$\Rightarrow \quad 27m^2 = 9$$

$$\Rightarrow \quad m^2 = \frac{1}{3} \quad \Rightarrow \quad m = \pm\frac{1}{\sqrt{3}}$$

Putting the value of $m = \frac{1}{\sqrt{3}}$ in (ii), we get

$$\frac{1}{\sqrt{3}}x - y - \frac{6}{\sqrt{3}} = 0 \quad \Rightarrow \quad x - \sqrt{3}\,y - 6 = 0 \qquad \text{... (iii)}$$

Putting the value of $m = -\frac{1}{\sqrt{3}}$ in (ii), we get

$$-\frac{1}{\sqrt{3}}x - y + \frac{6}{\sqrt{3}} = 0 \quad \Rightarrow -x - \sqrt{3}\,y + 6 = 0 \quad \Rightarrow x + \sqrt{3}\,y - 6 = 0 \qquad \text{... (iv)}$$

Hence, required equation of the lines are

$$x + \sqrt{3}\,y - 6 = 0, \quad x - \sqrt{3}\,y - 6 = 0.$$

Date: 12/10/2021

Q1. Find the equation of the line drawn through the point of intersection of the lines $4x - 3y + 7 = 0$ and $2x + 3y + 5 = 0$ and passing through the point $(-4, 5)$.

Q2. Find the equation of the line passing through the point of intersection of $x + 2y = 5$ and $x - 3y = 7$ and passing through the point $(2, -3)$.

Q3. Find the equation of the line through the intersection of $y + x = 9$ and $2x - 3y + 7 = 0$ and is perpendicular to the line $-3x + 2y - 5 = 0$.

Q4. A straight line passes through the point of intersection of the lines $x + 2y + 11 = 0$ and $3x - y + 5 = 0$ and makes an angle of 45° with the positive direction of the x-axis. Find its equation.

Q5. Find the equation of the straight line through the point of intersection of the lines $ax + by + c = 0$ and $a'x + b'y + c' = 0$ drawn parallel to x-axis.

Q6. Find equation of the line parallel to y-axis and drawn through the point of intersection of the lines $x - 7y + 5 = 0$ and $3x + y = 0$.

Q7. Find the equation of the line through the intersection of lines $3x + 4y = 7$ and $x - y + 2 = 0$ and whose slope is 5.

Q8. Find the equation of the line through the intersection of lines $x + 2y - 3 = 0$ and $4x - y + 7 = 0$ and which is parallel to the line $5x + 4y - 20 = 0$.

Q9. Find the equation of the line which passes through the point of intersection of the lines $2x + 3y + 5 = 0$ and $3x + 4y - 18 = 0$ and parallel to the line $5x + 2y + 9 = 0$.

Q10. Find the equation of the line passing through the intersection of the lines $4x + 7y - 3 = 0$ and $2x - 3y + 1 = 0$ that has equal intercepts on the axes.

Q11. Find the equation of the line through the intersection of the lines $3x + y - 9 = 0$ and $4x + 3y - 7 = 0$ and which is perpendicular to the line $5x - 4y + 1 = 0$.

Q12. Find the equation of the line through the intersection of $5x - 3y = 1$ and $2x + 3y - 23 = 0$ and perpendicular to the line whose equation is $5x - 3y - 1 = 0$.

Q13. Find the equation of the line which passes through the point of intersection of the lines $3x - 4y + 6 = 0$ and $4x - y - 5 = 0$ and cuts off equal intercepts from the axes.

Q14. Find the equation of the straight line which passes through the intersection of the straight lines $3x + 2y + 4 = 0$ and $x - y = 2$ and forms the triangle with the axes whose area is 8 square units.

Q15. Find the equations of the straight line passing through the intersection of lines $4x - 3y - 1 = 0$ and $2x - 5y + 3 = 0$ and equally inclined to the axes.

Date: 12/10/2021

S1. The equation of any line through the point of intersection of the given lines is of the form

$$(4x - 3y + 7) + k(2x + 3y + 5) = 0$$

$$\Rightarrow \quad (4 + 2k)x + (3k - 3)y + (5k + 7) = 0 \quad \ldots \text{(i)}$$

If it passes through the point (– 4, 5), we have:

$$(4 + 2k)(-4) + (3k - 3)\cdot 5 + (5k + 7) = 0 \quad [k \in R]$$

$$\Rightarrow \ -16 - 8k + 15k - 15 + 5k + 7 = 0 \ \Rightarrow \ 12k = 24 \ \Rightarrow \ k = 2$$

Substituting $k = 2$ in (i), we get:

$8x + 3y + 17 = 0$, which is the required equation.

S2. We have,

$$x + 2y = 5 \quad \ldots \text{(i)}$$

and $$x - 3y = 7 \quad \ldots \text{(ii)}$$

The equation of any line through the point of intersection of given lines (i) and (ii) is

$$(x + 2y - 5) + k(x - 3y - 7) = 0 \quad \ldots \text{(iii)}$$

If the point (2, – 3) lies on (iii), then

$$(2 + 2(-3) - 5) + k(2 - 3(-3) - 7) = 0$$

$$\Rightarrow \quad 2 - 6 - 5 + k(2 + 9 - 7) = 0 \ \Rightarrow \ -9 + 4k = 0$$

$$\Rightarrow \quad k = \frac{9}{4}$$

Substituting this value of k in equation (iii), we get

$$(x + 2y - 5) + \frac{9}{4}(x - 3y - 7) = 0 \ \Rightarrow \ 4(x + 2y - 5) + 9(x - 3y - 7) = 0$$

$$\Rightarrow \quad 4x + 8y - 20 + 9x - 27y - 63 = 0 \ \Rightarrow \ 13x - 19y - 83 = 0.$$

S3. We have $$x + y - 9 = 0 \quad \ldots \text{(i)}$$

and $$2x - 3y + 7 = 0 \quad \ldots \text{(ii)}$$

The equation of any line through point of intersection of lines (i) and (ii) is

$$(x + y - 9) + k(2x - 3y + 7) = 0$$

or $$(1 + 2k)x + (1 - 3k)y + (-9 + 7k) = 0 \quad \ldots \text{(iii)}$$

Slope of eq. (iii), $m_1 = \frac{-(1+2k)}{(1-3k)}$ Again $-3x + 2y - 5 = 0$ ... (iv)

Slope of eq. (iv), $m_2 = \frac{3}{2}$. As (iii) and (iv) are $\perp$ to each other,

$\therefore$ $m_1 \times m_2 = -1$

$\therefore$ $\frac{-(1+2k)}{1-3k} \times \frac{3}{2} = -1 \Rightarrow 3 + 6k = 2 - 6k \Rightarrow 1 = -12k$

$\Rightarrow$ $\boxed{k = \frac{-1}{12}}$

Now, puting value of k in eq. (iii), we get

$$(x + y - 9) - \frac{1}{12}(2x - 3y + 7) = 0 \Rightarrow 12x + 12y - 108 - 2x + 3y - 7 = 0$$

$\Rightarrow$ $10x + 15y - 115 = 0 \Rightarrow \boxed{2x + 3y - 23 = 0}$

S4. Let $x + 2y + 11 = 0$ ... (i)

and $3x - y + 5 = 0$ ... (ii)

The equation of the family of lines passing through the intersection of (i) and (ii) is

$(x + 2y + 11) + k(3x - y + 5) = 0$ ... (iii)

$\Rightarrow$ $(1 + 3k)x + (2 - k)y + (11 + 5k) = 0$

Slope, $m = \frac{-(1+3k)}{2-k}$

But this line makes an angle of 45° with the x-axis.

$\therefore$ $m = \tan 45° = 1$

$\therefore$ $\frac{-(1+3k)}{2-k} = 1 \Rightarrow 1 + 3k = k - 2 \Rightarrow 2k = -3 \Rightarrow \boxed{k = \frac{-3}{2}}$

Put this value of k in eq. (iii), we get

$$(x + 2y + 11) - \frac{3}{2}(3x - y + 5) = 0 \Rightarrow 2x + 4y + 22 - 9x + 3y - 15 = 0$$

$\Rightarrow$ $-7x + 7y + 7 = 0 \Rightarrow \boxed{x - y - 1 = 0}$

S5. Let, $ax + by + c = 0$ ... (i)

and $a'x + b'y + c' = 0$ ... (ii)

The equation of any line through the intersection of the given lines (i) and (ii) is of the form

$$(ax + by + c) + k(a'x + b'y + c') = 0 \quad \text{... (iii)}$$

$\Rightarrow \quad (a + a'k)x + (b + b'k)y + (c + c'k) = 0$

$\therefore \qquad \text{Slope} = \dfrac{-(a + a'k)}{(b + b'k)}$

For a line parallel to x-axis, slope = 0

$\therefore \qquad \dfrac{-(a + a'k)}{b + b'k} = 0 \Rightarrow a + a'k = 0 \Rightarrow \boxed{k = \dfrac{-a}{a'}}$

Putting this value of k in eq. (iii), we get

$$(ax + by + c) + \frac{a}{a'}(a'x + b'y + c') = 0$$

$\Rightarrow \quad a \cdot a'x + b \cdot a'y + c \cdot a' - a \cdot a'x - a \cdot b'y - a \cdot c' = 0$

$\Rightarrow \quad \boxed{(a' \cdot b - a \cdot b')y + (a'c - ac') = 0}$.

S6. Given, $x - 7y + 5 = 0$ ... (i)

and $3x + y = 0$

$\Rightarrow \quad y = -3x$ ... (ii)

Now, Eq. (i), gives

$$x - 7(-3x) + 5 = 0$$

$$x + 21x + 5 = 0$$

$\Rightarrow \quad x = \dfrac{-5}{22}$

Therefore, $y = -3\left(\dfrac{-5}{22}\right) = \dfrac{15}{22}$

Thus, point of intersection is $\left(\dfrac{-5}{22}, \dfrac{15}{22}\right)$

Any line parallel to y-axis is $x = k$

or $\dfrac{-5}{22} = k$,

Thus, the required line is $x = \dfrac{-5}{22}$.

S7. The given lines are 3x + 4y – 7 = 0 and x – y + 2 = 0

The equation of any line through the point of of intersection of the given lines is of the form.

$$(3x + 4y - 7) + k(x - y + 2) = 0 \quad \text{... (i)}$$

$\Rightarrow \quad (3 + k)x + (4 - k)y + (2k - 7) = 0$

$$(4 - k)y = -(3 + k)x + (7 - 2k)$$

$$y = \frac{-(3+k)}{4-k}x + \frac{(7-2k)}{(4-k)}$$

$$y = \frac{(k+3)}{(k-4)}x + \frac{(7-2k)}{(4-2k)}$$

Slope of this line is $\frac{(k+3)}{(k-4)}$.

$\therefore \quad \frac{k+3}{k-4} = 5 \Rightarrow k + 3 = 5k - 20 \Rightarrow 4k = 23 \Rightarrow k = \frac{23}{4}$

Substituting $k = \frac{23}{4}$ in (i), we get:

$$(3x + 4y - 7) + \frac{23}{4}(x - y + 2) = 0$$

$\Rightarrow \quad 4(3x + 4y - 7) + 23(x - y + 2) = 0$

$\Rightarrow \quad 35x - 7y + 18 = 0,$ which is the required equation.

S8. $5x + 4y - 20 = 0 \Rightarrow y = \frac{-5}{4}x + 5$

$\therefore$ slope of the given line = $\frac{-5}{4}$

and slope of the required line = $\frac{-5}{4}$

Now, the equation of any line through the intersection of the given lines is of the form

$(x + 2y - 3) + k(4x - y + 7) = 0$ … (i)

$\Rightarrow \quad (1 + 4k)x + (2 - k)y + (7k - 3) = 0$

Þ $\quad (2 - k)y = -(1 + 4k)x + (3 - 7k)$

$\Rightarrow \quad y = \frac{-(1+4k)}{(2-k)}x + \frac{(3-7k)}{(2-k)}$

$\Rightarrow \quad y = \frac{(1+4k)}{(k-2)}x + \frac{(3-7k)}{(2-k)}$

Slope of this line = $\frac{(1+4k)}{(k-2)}$

$\therefore \quad \frac{(1+4k)}{(k-2)} = \frac{-5}{4} \Leftrightarrow 4 + 16k = -5k + 10$

$$\Leftrightarrow 21k = 6 \quad \Leftrightarrow \quad k = \frac{6}{21} = \frac{2}{7}$$

Substituting, $k = \frac{2}{7}$ in (i), we get:

$$(x + 2y - 3) + \frac{2}{7}(4x - y + 7) = 0$$

$\Rightarrow \quad (7x + 14y - 21) + (8x - 2y + 14) = 0$

$\Rightarrow \quad 15x + 12y - 7 = 0$, which is the required equation.

S9. We have, $\quad 2x + 3y + 5 = 0 \quad$ ... (i)

and $\quad 3x + 4y - 18 = 0 \quad$ ... (ii)

The equation of any line through the point of intersection of given lines (i) and (ii) is

$(2x + 3y + 5) + k(3x + 4y - 18) = 0$

$\Rightarrow \quad (2 + 3k)x + (3 + 4k)y + (5 - 18k) = 0 \quad$ ... (iii)

Slope of line (iii), $\quad m_1 = \frac{-(2+3k)}{3+4k}$, Again $5x + 2y + 9 = 0 \quad$ ... (iv)

Slope of line (iv) $\quad m_2 = \frac{-5}{2}$. For lines (iii) and (iv) to be parallel, $m_1 = m_2$

$\therefore \quad \frac{-(2+3k)}{3+4k} = \frac{-5}{2} \quad \Rightarrow \quad 4 + 6k = 15 + 20k$

$\Rightarrow \quad -14k = 11 \quad \Rightarrow \quad \boxed{k = \frac{-11}{14}}$

Putting the value of k in (iii), we get

$$(2x + 3y + 5) - \frac{11}{14}(3x + 4y - 18) = 0$$

$\Rightarrow \; 28x + 42y + 70 - 33x - 44y + 198 = 0 \quad \Rightarrow \quad -5x - 2y + 268 = 0$

$\Rightarrow \quad \boxed{5x + 2y = 268}$.

S10. We have,

$4x + 7y - 3 = 0 \quad$ ... (i)

and $\quad 2x - 3y + 1 = 0 \quad$ ... (ii)

The equation of the family of lines passing through the point of intersection of (i) and (ii) is

$(4x + 7y - 3) + k(2x - 3y + 1) = 0 \quad$ ... (iii)

$\Rightarrow \quad (4 + 2k)x + (7 - 3k)y + k - 3 = 0 \quad$ ... (iv)

Since (iv) cuts off equal intercepts

$\Rightarrow \quad (2k + 4)x + (7 - 3k)y = 3 - k \quad \Rightarrow \quad \frac{(2k+4)x}{3-k} + \frac{(7-3k)y}{3-k} = 1$

$\Rightarrow \quad \frac{x}{\frac{3-k}{2k+4}} + \frac{y}{\frac{3-k}{7-3k}} = 1$ which is in the intercept form.

But it is given that intercepts are equal.

So, $\quad \frac{3-k}{2k+4} = \frac{3-k}{7-3x} \quad \Rightarrow \quad \frac{1}{2k+4} = \frac{1}{7+3k}$

$\Rightarrow \quad 2k + 4 = 7 - 3k \quad \Rightarrow \quad 5k = 3 \quad \Rightarrow \quad k = \frac{3}{5}$

Substituting $k = \frac{3}{5}$ in (iii), we get

$(4x + 7y - 3) + \frac{3}{5}(2x - 3y + 1) = 0$

$\Rightarrow \; 20x + 35y - 15 + 6x - 9y + 3 = 0 \quad \Rightarrow \quad 26x + 26y - 12 = 0$

$\Rightarrow \quad 13(x + y) - 6 = 0.$

S11. $5x - 4y + 1 = 0 \quad \Leftrightarrow \; y = \frac{5}{4}x + \frac{1}{4}$

Slope of the given line is $\frac{5}{4}$

Now, the equation of any line through the intersection of the given lines of the form

$(3x + y - 9) + k(4x + 3y - 7) = 0$ ... (i)

$\Rightarrow \; (3 + 4k)x + (1 + 3k)y - (9 + 7k) = 0$

$\Rightarrow \quad (1 + 3k)y = -(3 + 4k)x + (9 + 7k)$

$\Rightarrow \quad y = -\frac{(3+4k)}{(1+3k)}x + \frac{(9+7k)}{(1+3k)}$ ... (ii)

Let m be the slope of the line perpendicular to the required line.

Then, $\quad m \times \frac{5}{4} = -1 \Rightarrow \quad m = \frac{-4}{5}$

$\therefore$ we must have $\quad \frac{-(3+4k)}{(1+3k)} = \frac{-4}{5}$

$\Rightarrow \quad 15 + 20k = 4 + 12k \quad \Rightarrow \quad 8k = -11 \quad \Rightarrow \quad k = \frac{-11}{8}$

Substituting $k = \frac{-11}{8}$ in (i), we get

$$(3x + y - 9) - \frac{11}{8}(4x + 3y - 7) = 0$$

$\Rightarrow \quad (24x + 8y - 72) - 44x - 33y + 77 = 0$

$\Rightarrow \quad 20x + 25y - 5 = 0 \quad \Rightarrow \quad 4x + 5y - 1 = 0.$

S12. We have,

$5x - 3y - 1 = 0$ ... (i)

and $2x + 3y - 23 = 0$ ... (ii)

The equation of any line through the line of intersection of the given lines (i) and (ii) is of the form

$(5x - 3y - 1) + k(2x + 3y - 23) = 0$ ... (iii)

$(5 + 2k)x + 3(k - 1)y - (23k + 1) = 0$

$$y = \frac{(5+2k)}{3(k-1)}x + \frac{(23k+1)}{3(k-1)} \quad \text{... (iv)}$$

$$\text{Slope of the line} = -\frac{5+2k}{3(k-1)}$$

Again $5x - 3y - 1 = 0 \quad \Rightarrow \quad y = \frac{5}{3}x - \frac{1}{3}$ ... (v)

$$\text{Slope} = \frac{5}{3}$$

Since (iv) is perpendicular to (v), we have,

$$-\frac{(5+2k)}{3(k-1)} \times \left(\frac{5}{3}\right) = -1 \qquad [m_1 m_2 = -1]$$

$\Rightarrow \quad \frac{25+10k}{9k-9} = 1 \quad \Rightarrow \quad 25 + 10k = 9k - 9$

$\Rightarrow \quad k = -34$

Substituting the value of k in (iii), we get

$(5x - 3y - 1) - 34(2x + 3y - 23) = 0$

$\Rightarrow \quad 5x - 68x - 3y - 102y - 1 + 782 = 0$

$\Rightarrow \quad 63x + 105y - 781 = 0.$

S13. Let $3x - 4y + 6 = 0$ ... (i)

and $4x - y - 5 = 0$ ... (ii)

The equation of any line through the intersection of lines (i) and (ii) is of the form

$$(3x - 4y + 6) + k(4x - y - 5) = 0 \qquad \text{... (iii)}$$

$\Rightarrow \quad (3 + 4k)x + (-4 - k)y + (6 - 5k) = 0$

$\Rightarrow \quad (3 + 4k)x - (4 + k)y = (5k - 6)$

$$\Rightarrow \quad \frac{x}{\frac{5k-6}{3+4k}} + \frac{y}{\frac{5k-6}{-(4+k)}} = 1$$

But it is given that the intercepts are equal.

$$\therefore \quad \frac{5k-6}{3+4k} = \frac{5k-6}{-(4+k)} \Rightarrow 3 + 4k = -4 - k$$

$$\Rightarrow \quad 7 = -5k \Rightarrow \boxed{k = \frac{-7}{5}}$$

Putting the value of k in eq. (iii), we get

$$(3x - 4y + 6) - \frac{7}{5}(4x - y - 5) = 0$$

$\Rightarrow \quad 15x - 20y + 30 - 28x + 7y + 35 = 0 \Rightarrow -13x - 13y + 65 = 0$

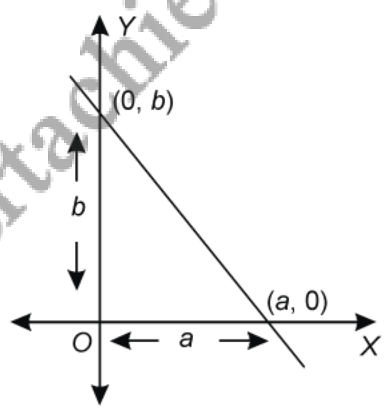
$\Rightarrow \quad \boxed{x + y - 5 = 0}$.

S14. Let, $3x + 2y + 4 = 0$ ... (i)

and $x - y - 2 = 0$ ... (ii)

The equation of any line through the intersection of lines (i) and (ii) is of the form

$$(3x + 2y + 4) + k(x - y - 2) = 0 \qquad \text{... (iii)}$$



$\Rightarrow \quad (3 + k)x + (2 - k)y + (4 - 2k) = 0$

$$\Rightarrow \quad \frac{(3+k)x}{-(4-2k)} + \frac{(2-k)y}{-(4-2k)} = 1$$

$$\Rightarrow \quad \frac{x}{\frac{2k-4}{3+k}} + \frac{y}{\frac{2k-4}{2-k}} = 1$$

$$\therefore \quad x\text{-intercept,} \quad a = \frac{2k-4}{3+k}$$

Now, Area of $\Delta = \frac{1}{2} \times a \times b \quad \therefore \quad \frac{1}{2} \times \frac{2k-4}{3+k} \times \frac{2k-4}{2-k} = 8$

$$\Rightarrow \quad \frac{4k^2 + 16 - 16k}{6 - k - k^2} = 16$$

$\Rightarrow \quad 4k^2 + 16 - 16k = 96 - 16k - 16k^2 \Rightarrow 20k^2 = 80$

$\Rightarrow$ $k^2 = 4 \Rightarrow \boxed{k = \pm 2}$ But

But if $k = 2$, $a = b = 0$. is not possible.

$\therefore$ $\boxed{k = -2}$. Put this value of k in eq. (iii), we get

$$(3x + 2y + 4) - 2(x - y - 2) = 0$$

$\Rightarrow$ $\boxed{x + 4y + 8 = 0}$.

S15. Equation of lines are

$$4x - 3y - 1 = 0 \quad \text{... (i)}$$

$$2x - 5y + 3 = 0 \quad \text{... (ii)}$$

The equation of the family of lines passing through the point of intersection of (i) and (ii) is

$$4x - 3y - 1 + k(2x - 5y + 3) = 0 \Rightarrow (4 + 2k)x + (-3 - 5k)y + 3k - 1 = 0$$

$$\text{Slope} = \frac{4 + 2k}{3 + 5k}$$

Since, the line is equally inclined to axes. Therefore two cases arise.

Case-I: $\tan 45° = \dfrac{4 + 2k}{3 + 5k}$

$1 = \dfrac{4 + 2k}{3 + 5k}$, $3 + 5k = 4 + 2k$, $3k = 1$, $k = \dfrac{1}{3}$

$\left(4 + \dfrac{2}{3}\right)x + \left(-3 - \dfrac{5}{3}\right)y + 1 - 1 = 0$

$\dfrac{14x}{3} - \dfrac{14}{3}y = 0$, $x = y$

Case-II: $\tan(-45°) = \dfrac{4 + 2k}{3 + 5k}$

$-1 = \dfrac{4 + 2k}{3 + 5k}$

$4 + 2k = -3 - 5k$

$7k = -7$, $k = -1$

$2x + 2y - 3 - 1 = 0$

$2x + 2y = 4$, $x + y = 2$